BENEFIT

Boosting the telecommunications engineer profile to meet modern society and industry needs

Sistemi sa više brzina (13M031SVB)
Deo 2

Master studije

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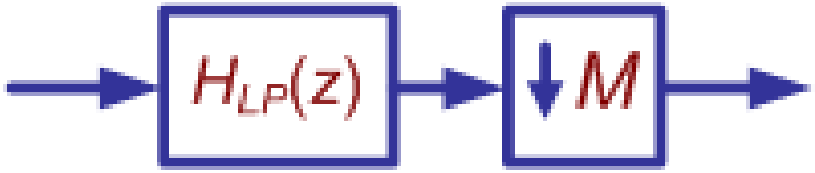
Decimacija

- Frekvencija odabiranja se smanjuje *M* puta
- Da ne bi bilo preklapanja u spektru, ulazni signal se, pre *downsampling*-a filtrira filtrom propusnikom niskih frekvencija, granične frekvencije

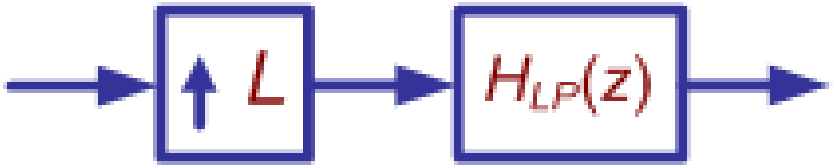
$$\omega_{LP} \leq \frac{\pi}{M}$$



Interpolacija

- Frekvencija odabiranja se povećava *L* puta
- Replike u spektru se potiskuju tako što se signal na izlazu iz bloka za *upsampling* filtrira filtrom propusnikom niskih frekvencija, granične frekvencije

$$\omega_{LP} \leq \frac{\pi}{L}$$



Promena frekvencije odabiranja *L*/*M* puta

- Kombinacija interpolacije s faktorom *L* i decimacije s faktorom *M*
- Filtar propusnik niskih frekvencija granične frekvencije

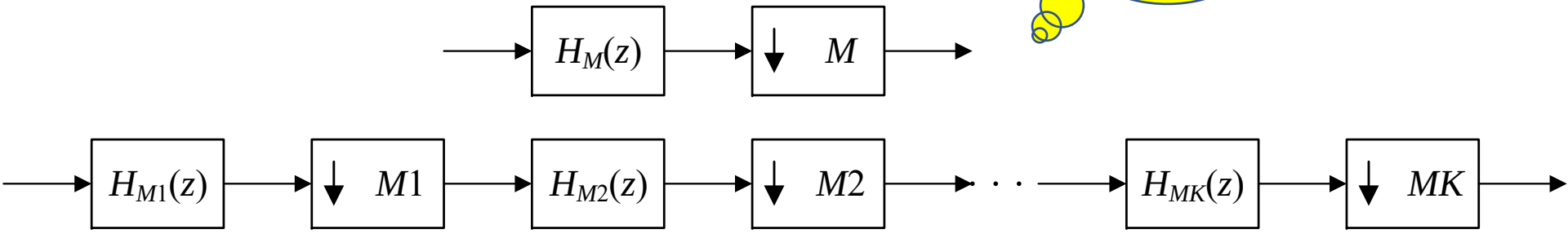
$$\omega_{LP} \le \min\left\{\frac{\pi}{M}, \frac{\pi}{L}\right\} = \frac{\pi}{\max\{M, L\}}$$

Višestepena decimacija/interpolacija

Umesto u jednom „koraku“, decimacija s faktorom *M* može se realizovati u nekoliko koraka, tako da je ukupan faktor decimacije:

$$M = \prod_{k=1}^{K} M_k$$

Višestepeni decimator



Višestepena decimacija/interpolacija

- Osnovna prednost je što su projektovanje i realizacija filtara jednostavniji (kasnije sledi primer)
- U nekim slučajevima, mogu se koristiti ponovljeni „standradizovani“ blokovi za male faktore decimacije (na primer 2, 3, 5…)

Decimacija/interpolacija s faktorom 2

- U slučaju decimacije ili interpolacije s faktorom 2,

$$\omega_{LP} \leq \frac{\pi}{2}$$

- problem projektovanja filtara se, za $\omega_{LP}=\pi/2$, može svesti na „*half-band*“ filtre, odnosno filtre koji dele frekvencijski opseg na dva dela
- *Half-band* filtri su, po pravilu, efikasni za realizaciju jer zbog simetrije frekvencijsog odziva imaju veliki broj nula u impulsnom odzivu

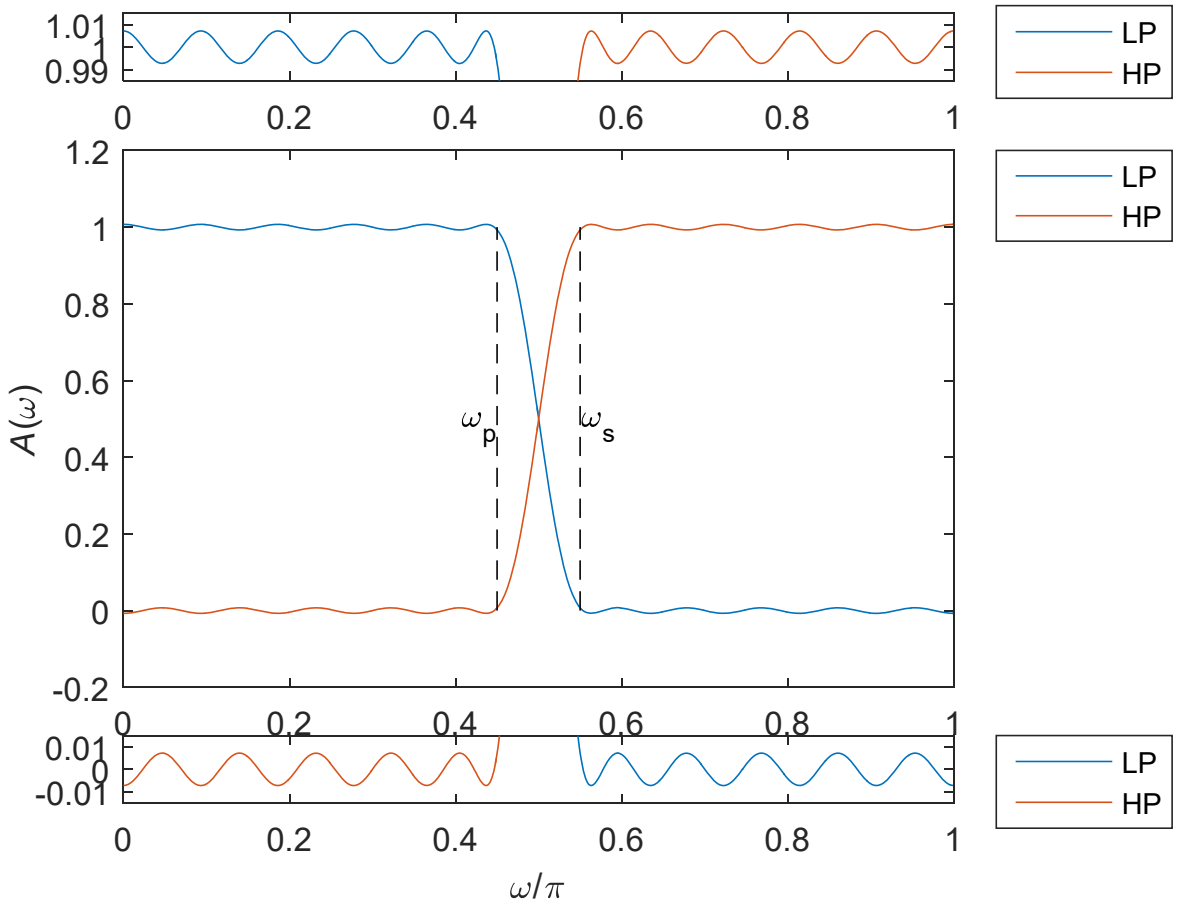
Half-band filtri/filtarski parovi

- Mogu biti FIR i IIR
- **L**(*ow*)**P**(*ass*) **H**(*igh*)**P**(*ass*) filtarski par deli opseg na dva dela
- LP i HP filtar nisu nezavisni, odnosno postoji relacija koja povezuje frekvencijske odzive LP i HP filtra
- Filtarski parovi su važni za projektovanje i realizaciju dvokanalnih banaka filtara
- Za decimaciju/interpolaciju nama su od interesa LP filtri

FIR *half-band* filtar

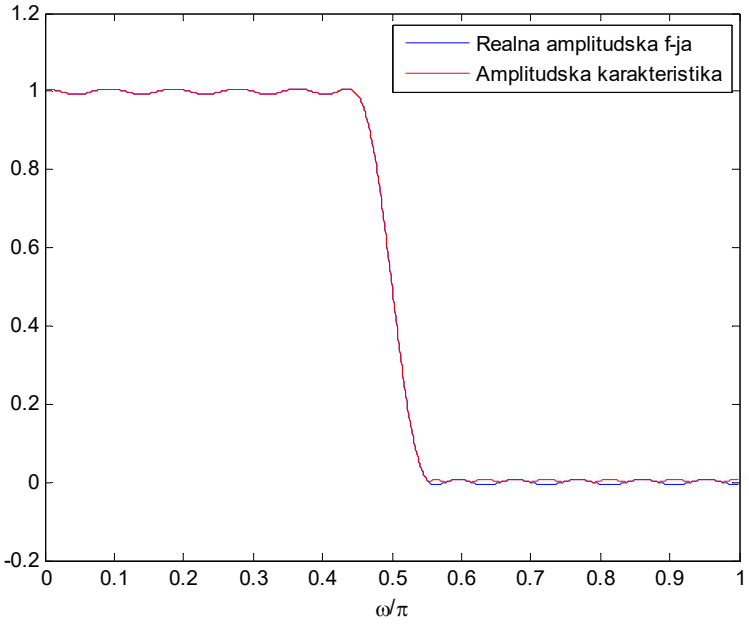
- $A(\pi/2)=0.5$
- $\delta=\delta_{PO}=\delta_{NPO}$
- $\pi/2-\omega_p=\omega_s-\pi/2$
- $\omega_p+\omega_s=\pi$

$\omega_p=0.45\pi$
$\delta=0.01$



Podsetnik

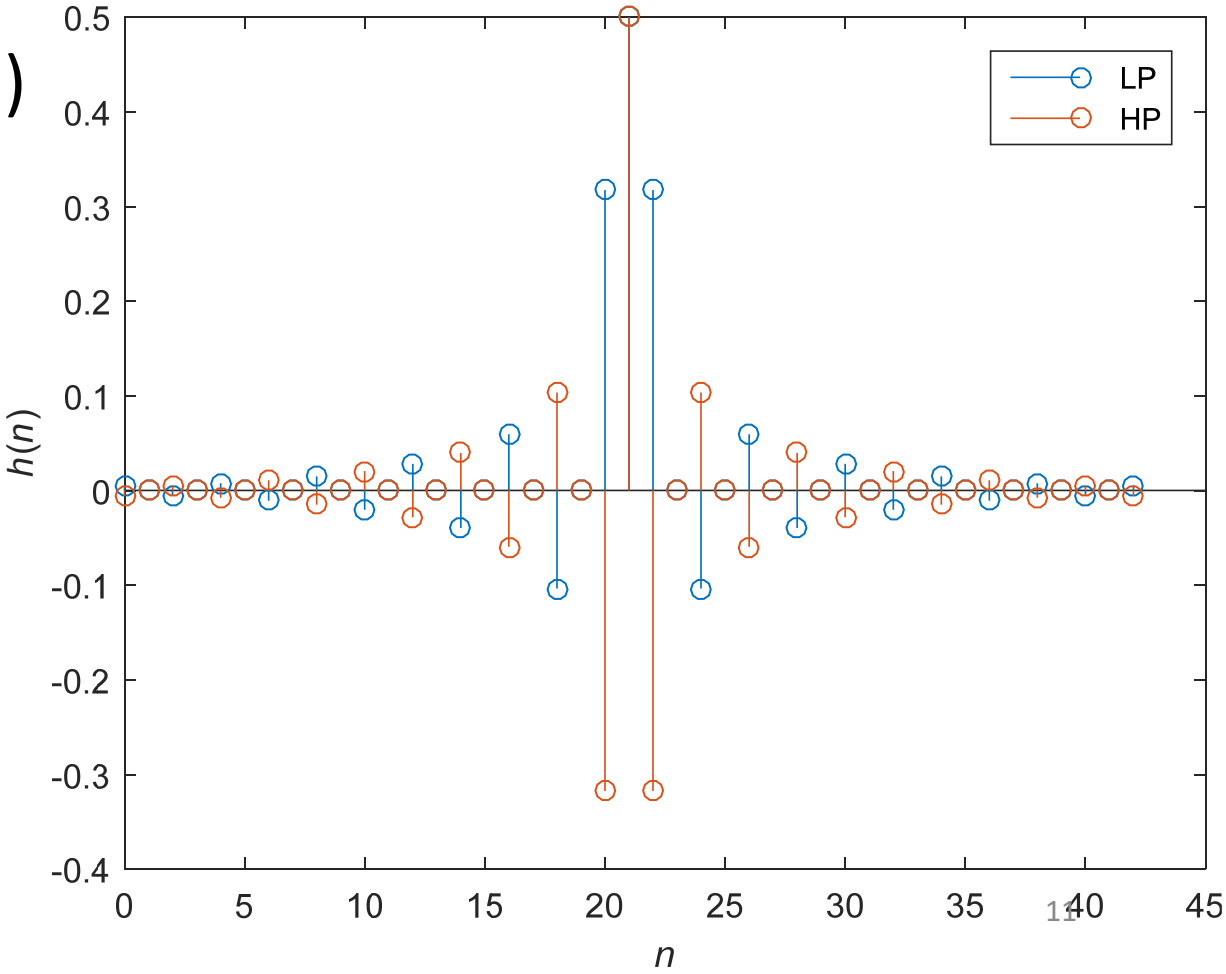
Kod FIR filtara linearne faze, umesto amplitudske karakteristike, može se posmatrati realna amplitudska funkcija (realna funkcija, može biti pozitivna i negativna)



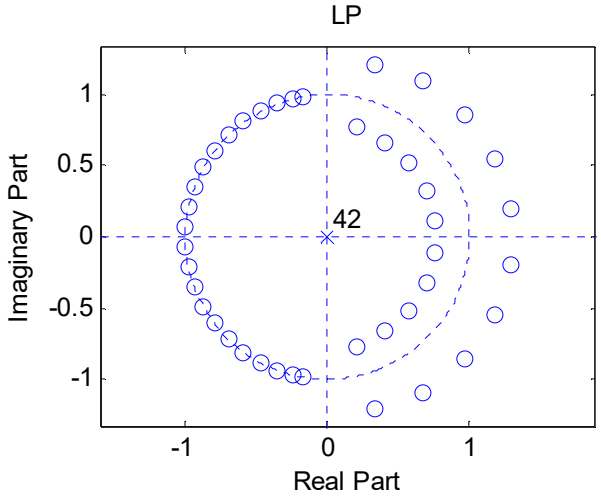
FIR *half-band* filtar

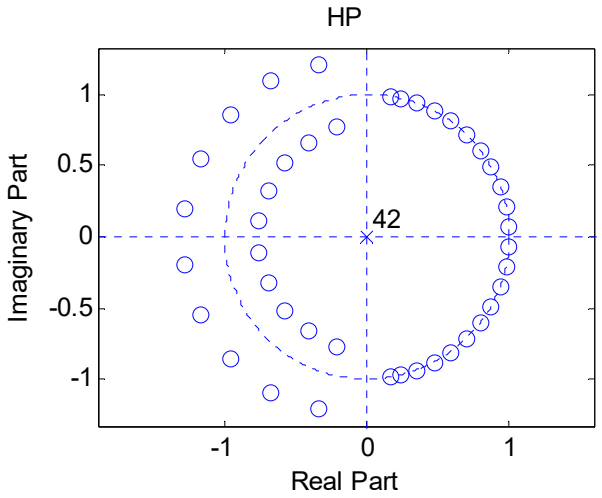
- red filtra *N* mora biti paran broj (zapravo $N=4m+2$)
- $h(N/2)=0.5$
- $h(N/2\pm 2l)=0$

N=42

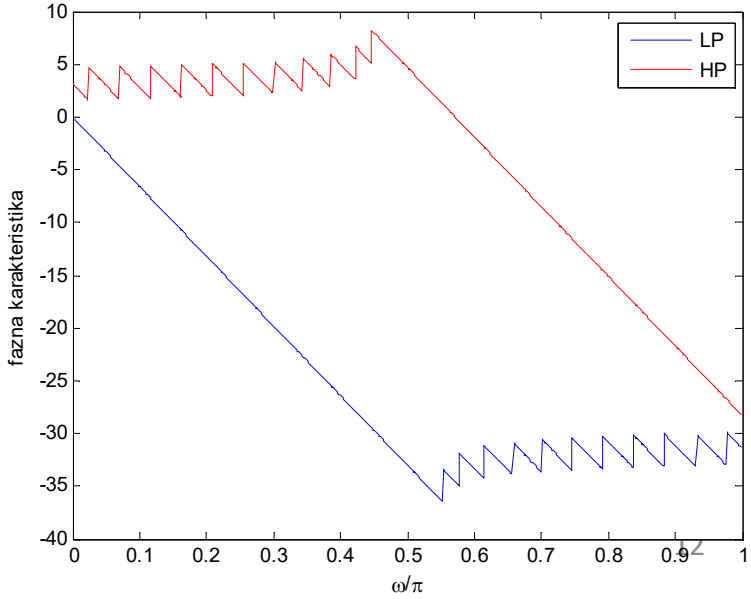


FIR *half-band* filtar





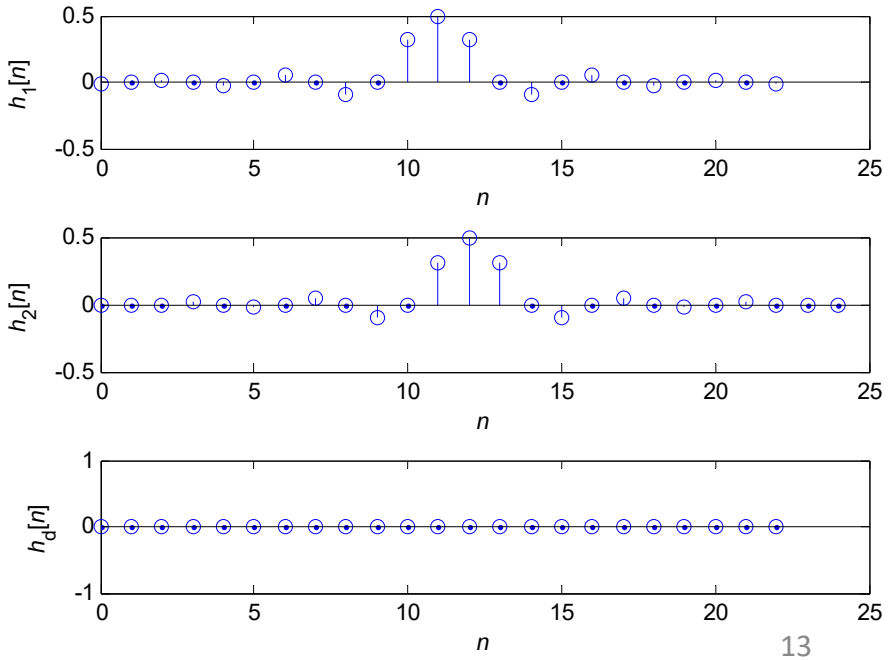
Linerana fazna k-ka



FIR *half-band* filtar

- Red filtra je oblika $4m+2$
- Za red filtra oblika $4m+4$ prvi i poslednji odbirak impulsnog odziva su nule

$N_1=22$
$N_2=24$
$h_d[n]=h_2[n+1]-h_1[n], \quad n=0,1,\ldots,N_1$



Prvih 5 odibraka impulsnih odziva h_1 (gornji red) i h_2

-0.0072 0 0.0136 0 -0.0263
0 -0.0072 0 0.0136 0

FIR *half-band* filtar

$$H(z)=\sum_{k=-N/2}^{N/2} h[N/2+k]z^{-(N/2+k)}=$$

$$=\sum_{l=-\lfloor N/4\rfloor}^{\lfloor N/4\rfloor} h[N/2+2l]z^{-(N/2+2l)}+\sum_{l=-\lfloor N/4\rfloor-1}^{\lfloor N/4\rfloor} h[N/2+2l+1]z^{-(N/2+2l+1)}$$

$$N=42$$

$$H(z)=\sum_{l=-10}^{10} h[21+2l]z^{-(21+2l)}+\sum_{l=-11}^{10} h[21+2l+1]z^{-(21+2l+1)}=$$

$$h[1]z^{-1}+h[3]z^{-3}+\cdots+h[19]z^{-19}+h[21]z^{-21}+h[23]z^{-23}+\cdots+h[41]z^{-41}+$$

$$h[0]z^{-0}+h[2]z^{-2}+\cdots+h[40]z^{-40}+h[42]z^{-42}$$

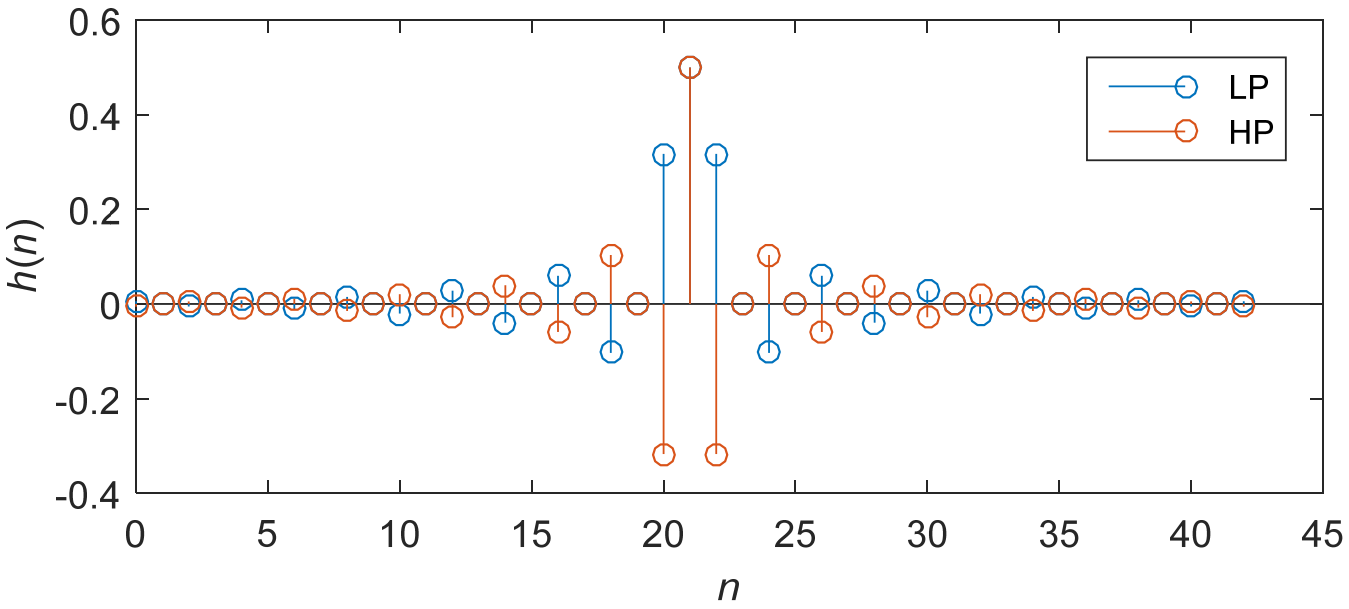
FIR *half-band* filtar

$$N = 42$$

$$= \sum_{l=-10}^{10} h[21+2l] z^{-(21+2l)} + \sum_{l=-11}^{10} h[21+2l+1] z^{-(21+2l+1)} =$$

$$h[1]z^{-1} + h[3]z^{-3} + \cdots + h[19]z^{-19} + h[21]z^{-21} + h[23]z^{-23} + \cdots + h[41]z^{-41} +$$

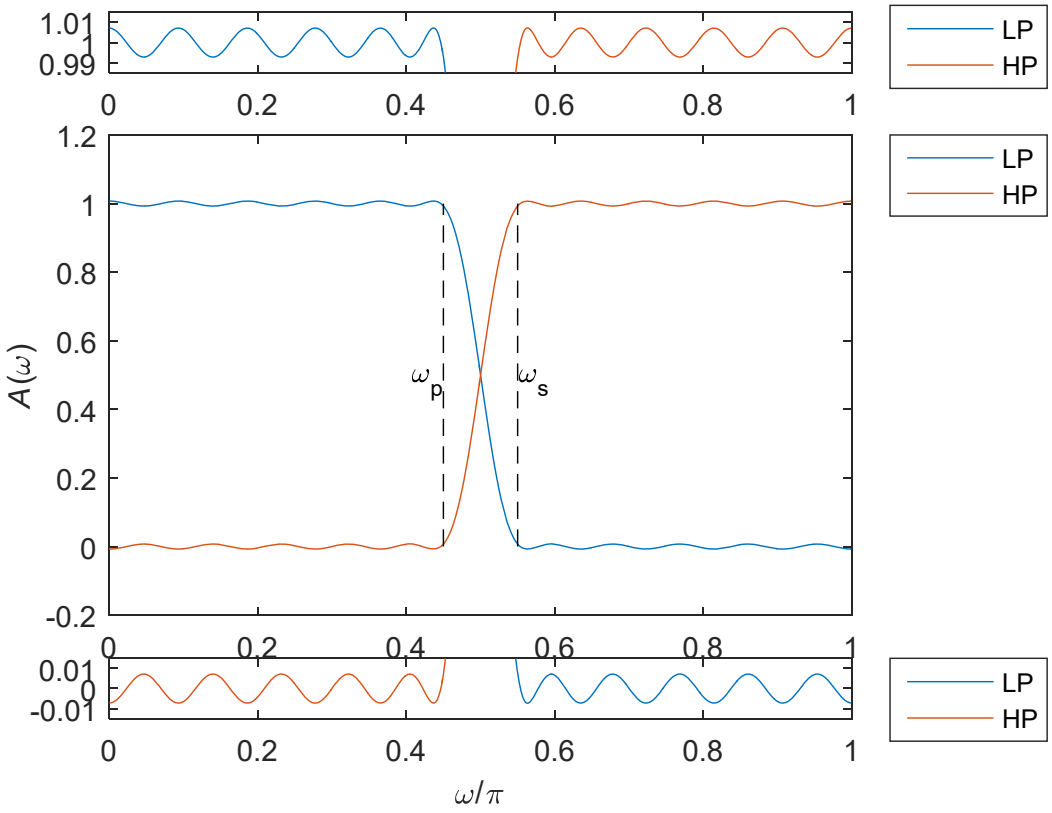
$$h[0]z^{-0} + h[2]z^{-2} + \cdots + h[40]z^{-40} + h[42]z^{-42}$$



FIR *half-band* filtar u MATLAB-u

```
wg=0.45;
dev=0.01;
% projektovanje LP half-band filtra
hlp=firhalfband('minorder',wg,dev);
N=length(hlp)-1;
hhp=-hlp; % HP
hhp(N/2+1)=0.5; %HP
```

$\omega_p=0.45\pi$



FIR *half-band* filtar efikasna realizacija

$$N = 42$$

$$H(z) = h[21]z^{-21} + \sum_{l=-11}^{10} h[21+2l+1]z^{-(21+2l+1)} =$$

$$h[21]z^{-21} +$$

$$h[0]z^{-0} + h[2]z^{-2} + \cdots + h[40]z^{-40} + h[42]z^{-42}$$

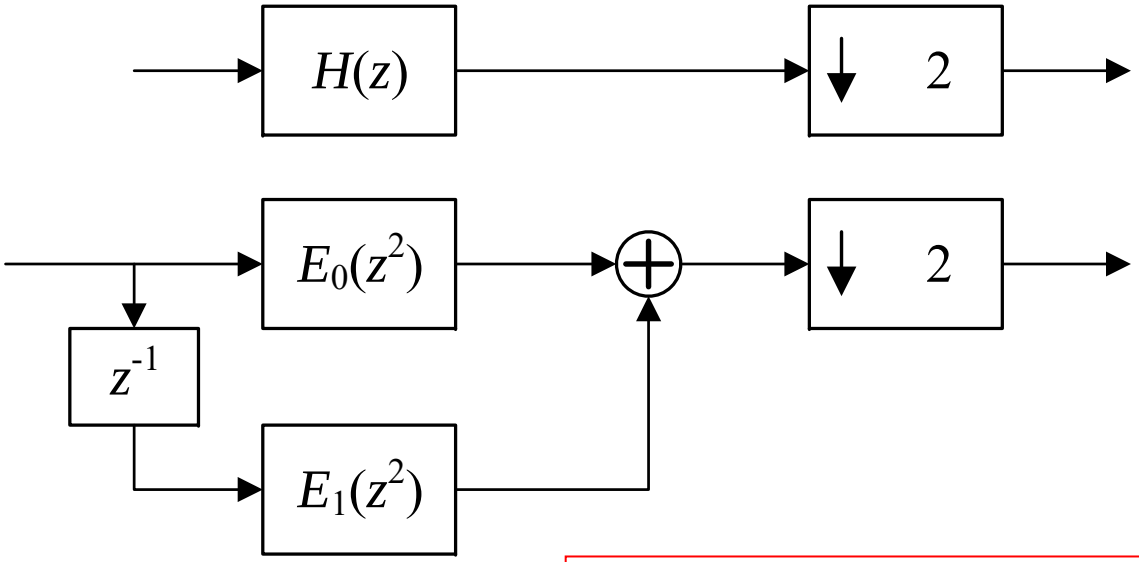
$$H(z) = H_1(z) + H_2(z)$$

$$H_1(z) = \sum_{m=0}^{N/2} h[2m]z^{-2m} = \sum_{m=0}^{N/2} h[2m]\left(z^2\right)^{-m} = E_0\left(z^2\right)$$

$$e_0[0] = h[0],\ e_0[1] = h[2],\ e_0[2] = h[4]\ldots$$

$$H_2(z) = h[21]z^{-21} = z^{-1}\left(h[21]\left(z^2\right)^{-10}\right) = z^{-1}E_1\left(z^2\right)$$

FIR *half-band* filtar kao decimacioni filtar



Polifazna dekompozicija za faktor decimacije 2

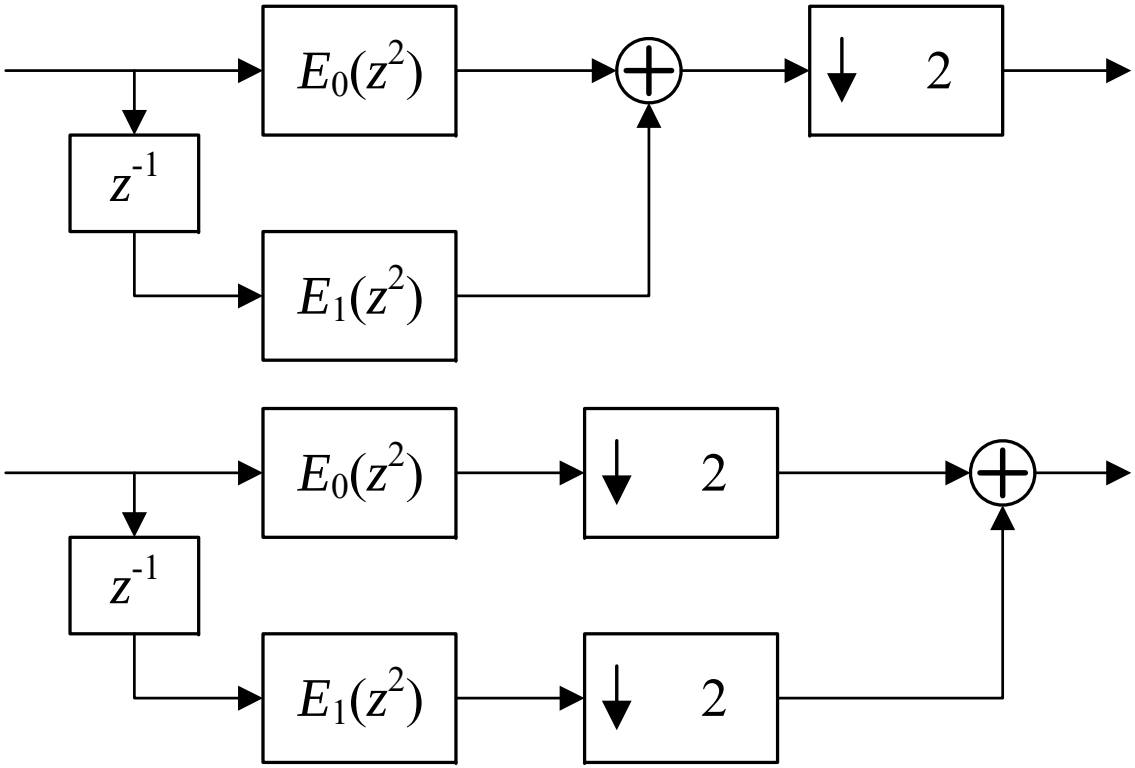
$$H(z) = H_1(z) + H_2(z)$$

$$H_1(z) = \sum_{m=0}^{N/2} h[2m] z^{-2m} = \sum_{m=0}^{N/2} h[2m] \left(z^2\right)^{-m} = E_0\left(z^2\right)$$

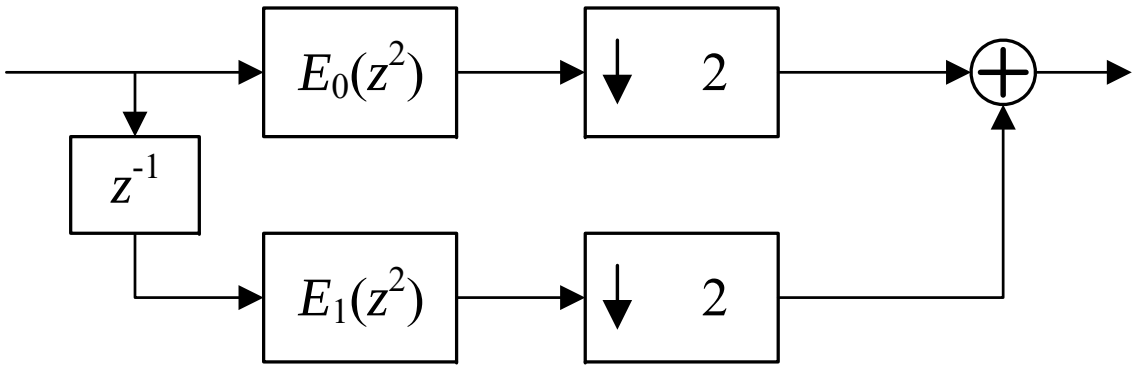
$$e_0[0] = h[0],\ e_0[1] = h[2],\ e_0[2] = h[4] \ldots$$

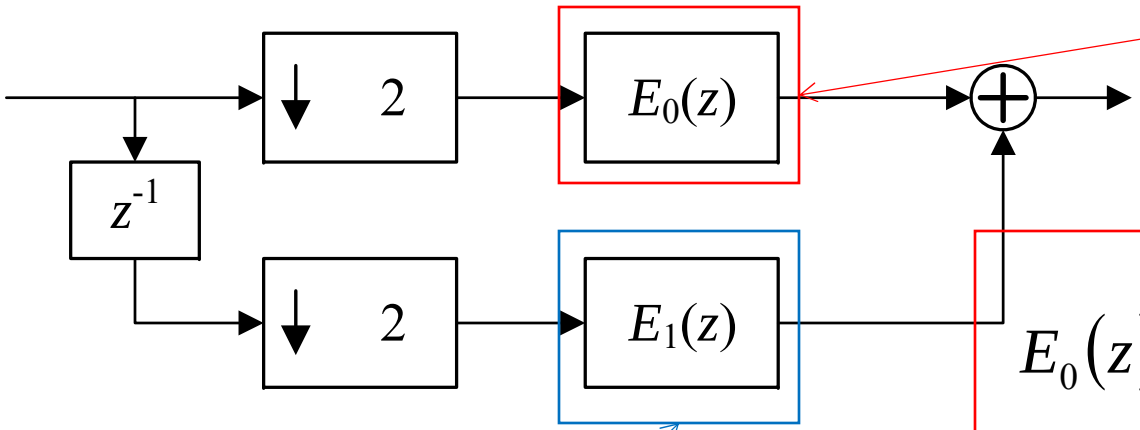
$$H_2(z) = h[21] z^{-21} = z^{-1}\left(h[21]\left(z^2\right)^{-10}\right) = z^{-1} E_1\left(z^2\right)$$

FIR *half-band* filtar kao decimacioni filtar



FIR *half-band* filtar kao decimacioni filtar





FIR filtar

$$E_0(z) = \sum_{k=0}^{N/2} e[k] z^{-k}$$

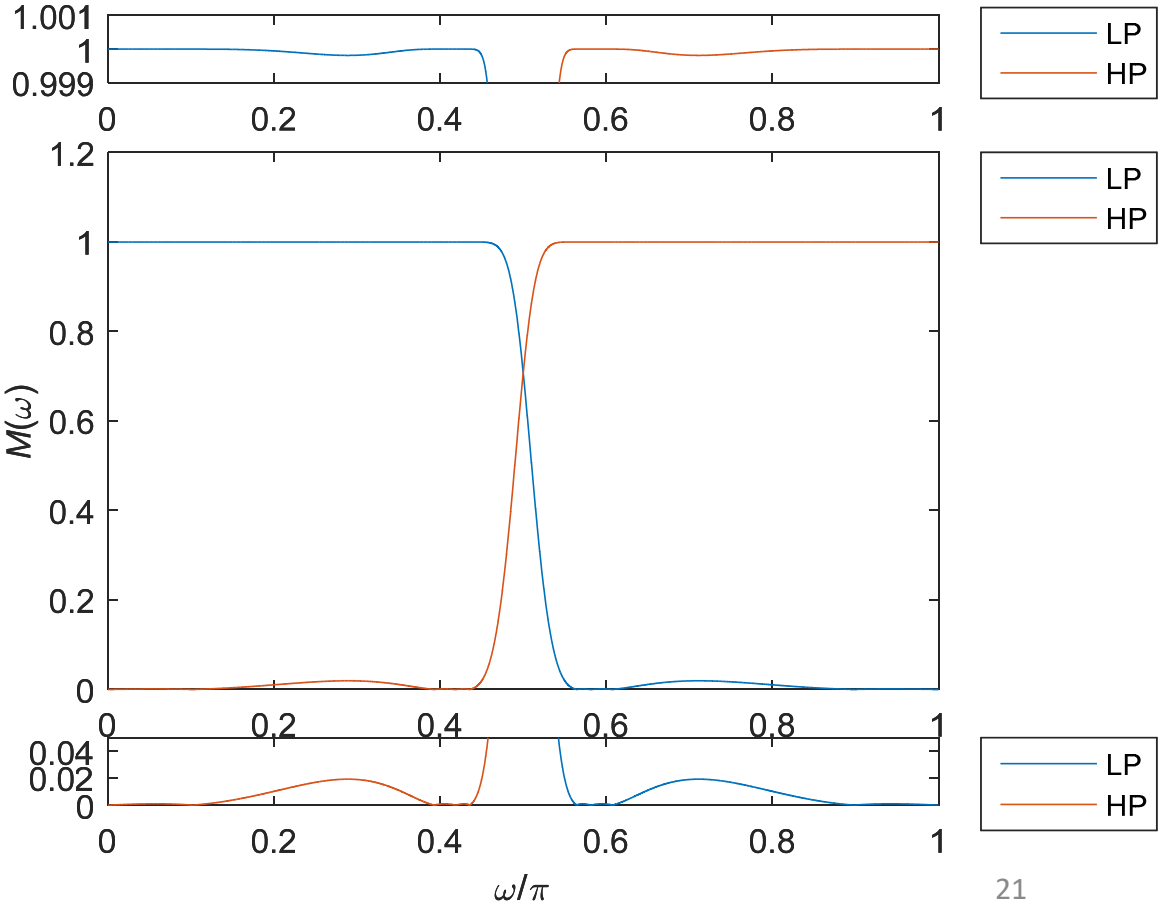
Kašnjenje i skaliranje s 0.5

$$E_1(z) = h[N/2] z^{-\frac{\frac{N}{2}-1}{2}} = 0.5 z^{-\frac{N-2}{4}}$$

IIR *half-band* filtar

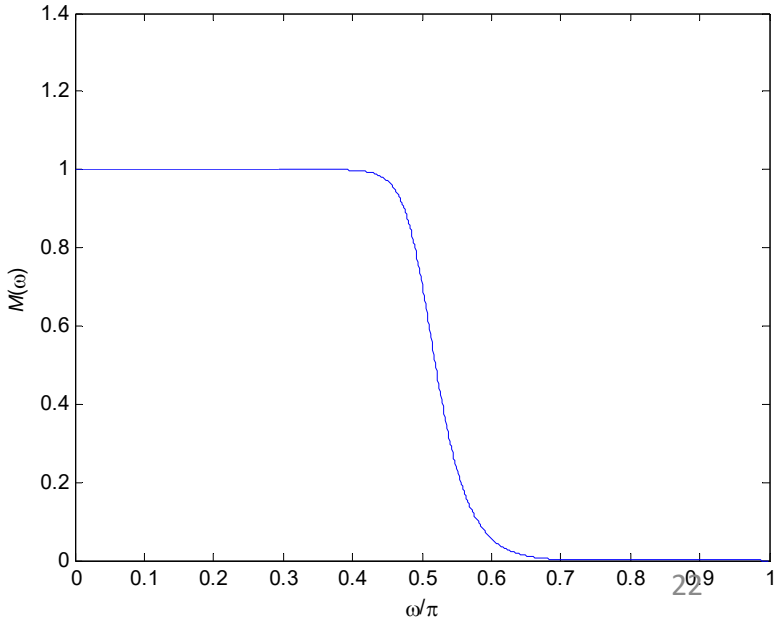
- $M^2(\pi/2)=0.5$
- $\pi/2-\omega_p= \omega_s -\pi/2$
- $\omega_p +\omega_s=\pi$

$\omega_p=0.45\pi$



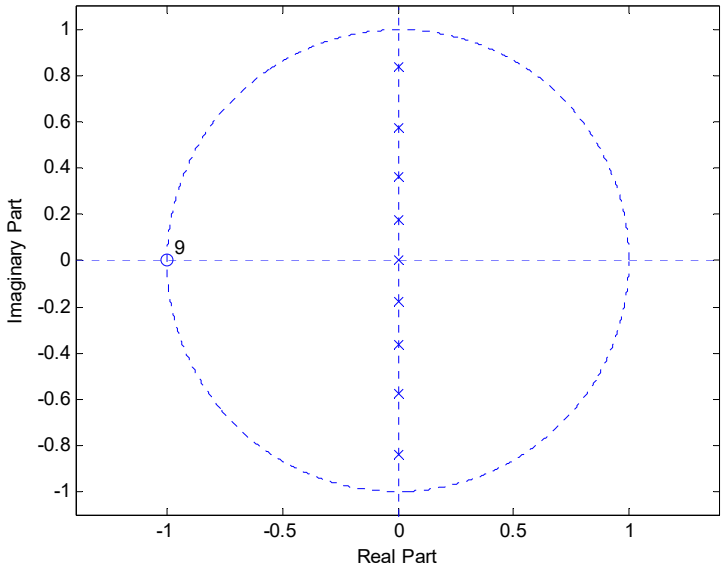
IIR *half-band* filtar

- 3 dB granična frekvencija je 0.5
- U MATLAB-u je pomoću funkcija koje smo već naučili, moguće projektovati *half-band Butterwort*-ov filtar

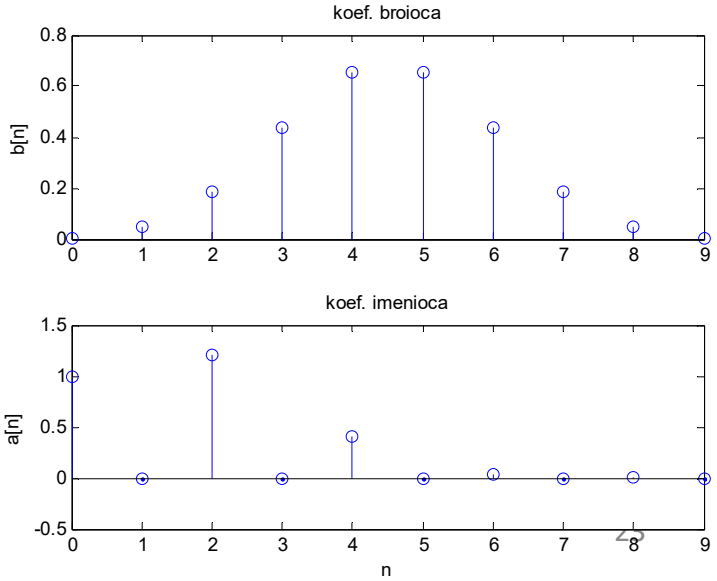


IIR *half-band* filtar

Svi polovi su na imaginarnoj osi

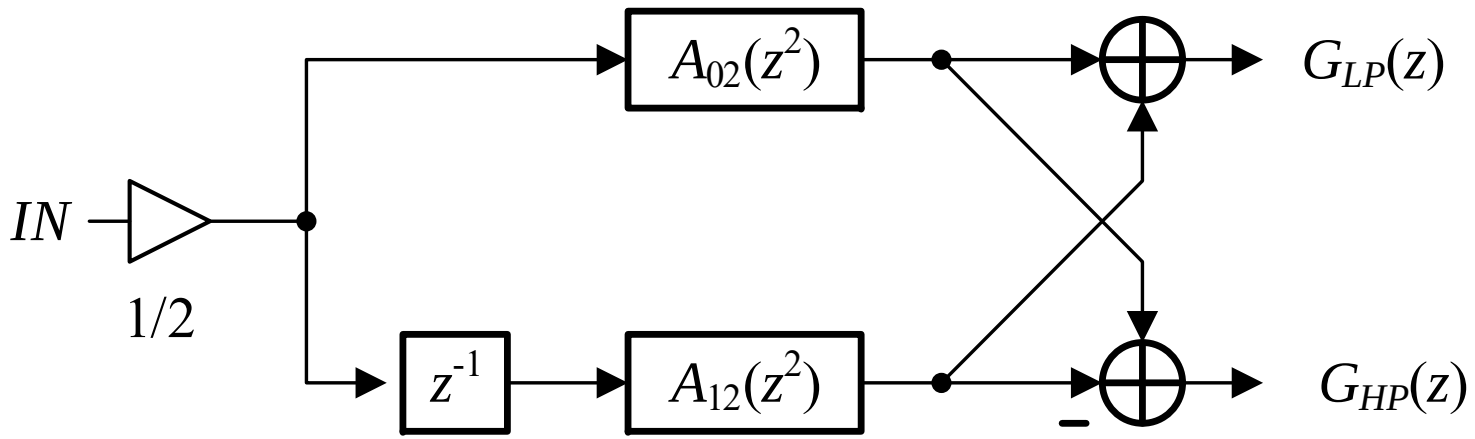


Svaki drugi koeficijent imenioca je 0



IIR *half-band* filtar

Realizacija preko paralelne veze svepropusniska



Detaljan postupak sledi kada budemo radili filtarske banke...

Polifazna dekompozicija FIR filtara

$$H(z)=\sum_{k=0}^{M-1} z^{-k} E_k\left(z^M\right)$$

$$E_k(z)=\sum_{n=0}^{\left\lfloor \frac{N}{M} \right\rfloor} h[Mn+k]z^{-n},\ \ k=0,\ldots M-1$$

Kada ovo koristimo u decimatorima/interpolatorima M je faktor promene učestanosti odabiranja

$primer\ \ M=3$

$$H(z)=h[0]z^{-0}+h[1]z^{-1}+h[2]z^{-2}+h[3]z^{-3}+h[4]z^{-4}+h[5]z^{-5}+\ldots$$

$$=h[0]z^{-0}+h[3]z^{-3}+h[6]z^{-6}+\ldots$$

$$+h[1]z^{-1}+h[4]z^{-4}+h[7]z^{-7}+\ldots$$

$$+h[2]z^{-2}+h[5]z^{-5}+h[8]z^{-8}+\ldots$$

$$H(z)=h[0]z^{-0}+h[3]z^{-3}+h[6]z^{-6}+\ldots$$

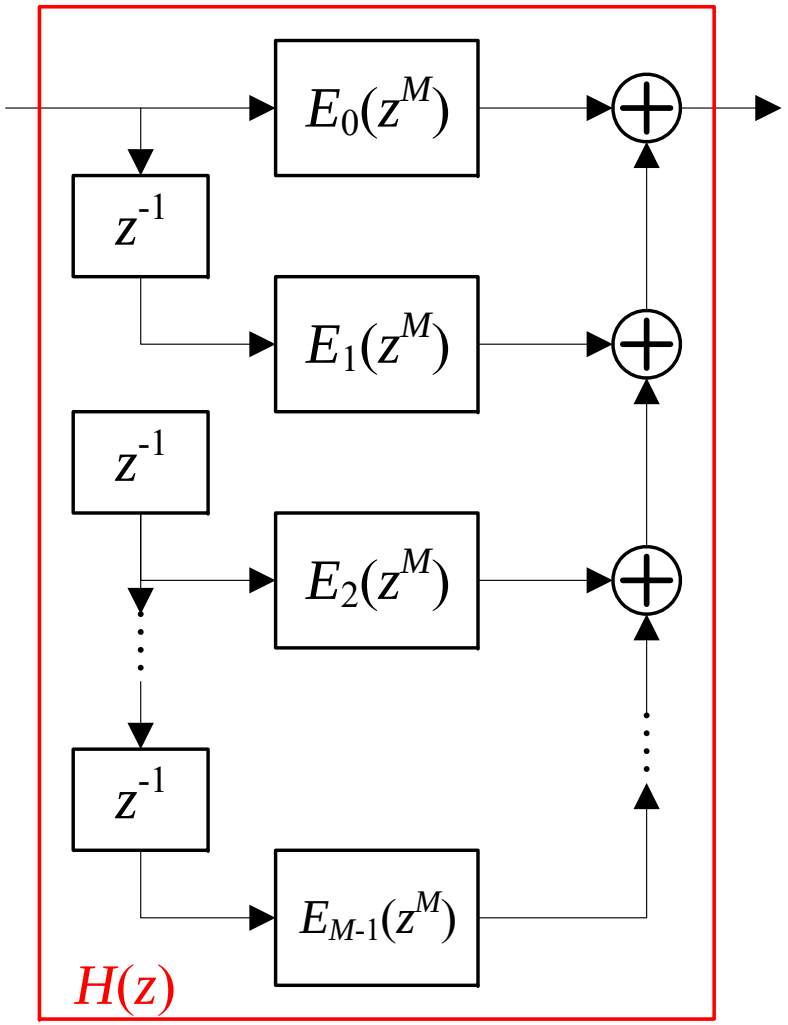
$$z^{-1}\left(h[1]z^{-0}+h[4]z^{-3}+h[7]z^{-6}\right)+\ldots$$

$$z^{-2}\left(h[2]z^{-0}+h[5]z^{-3}+h[8]z^{-6}\right)+\ldots$$

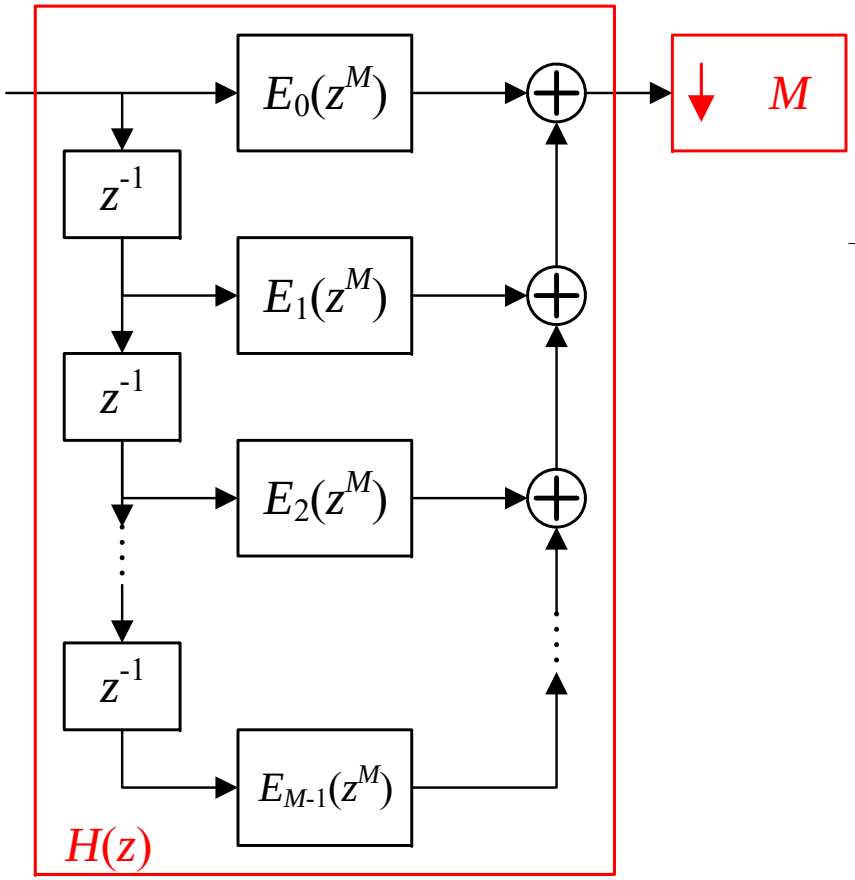
Polifazna dekompozicija

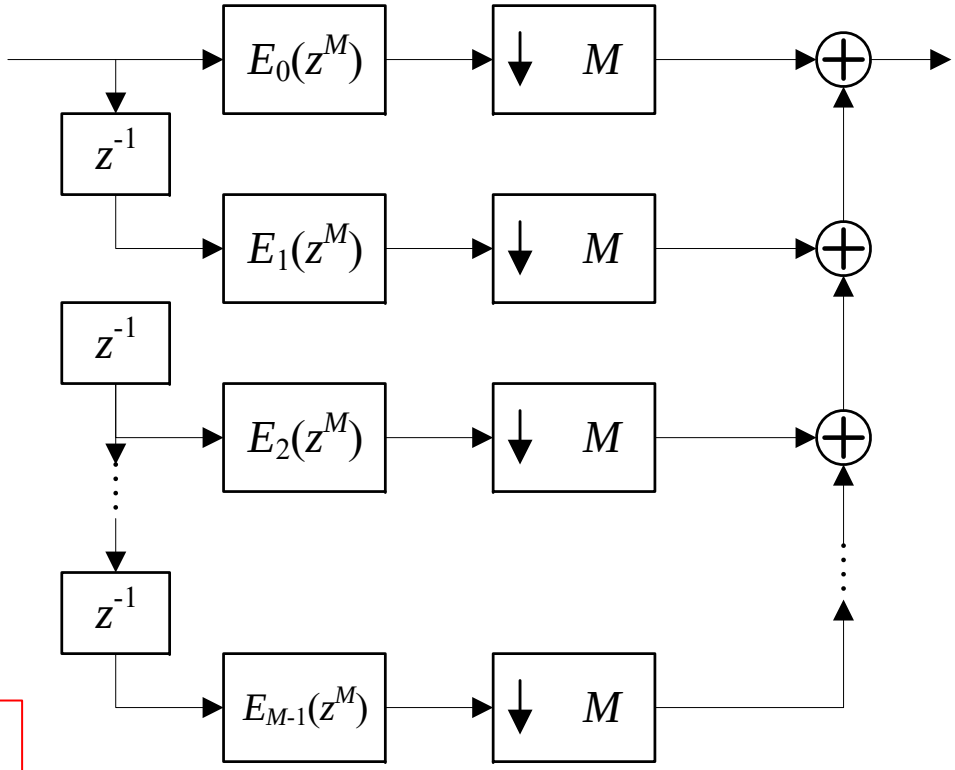
$$H(z)=\sum_{k=0}^{M-1} z^{-k} E_k\left(z^M\right)$$

$$E_k(z)=\sum_{n=0}^{\left\lfloor \frac{N}{M} \right\rfloor} h[Mn+k]z^{-n},\ \ k=0,\ldots M-1$$



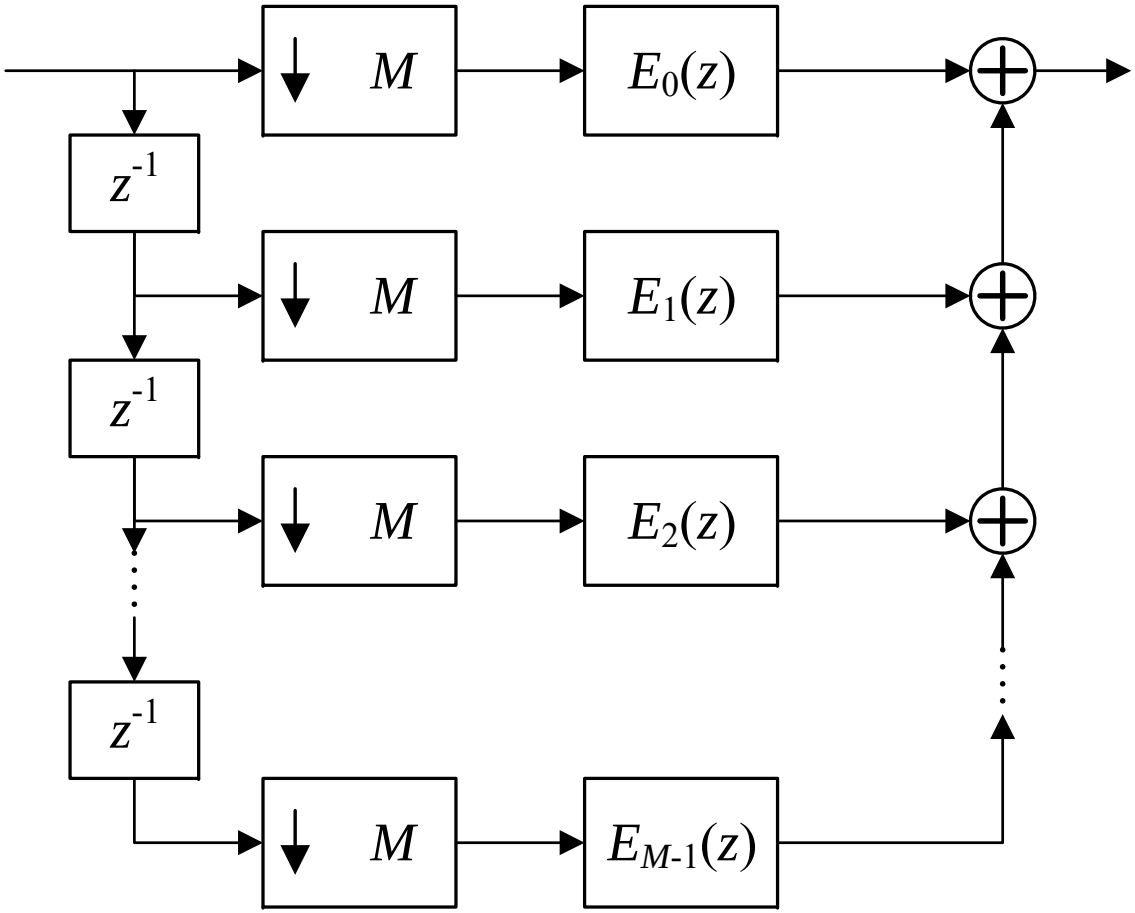
Polifazna dekompozicija - decimator





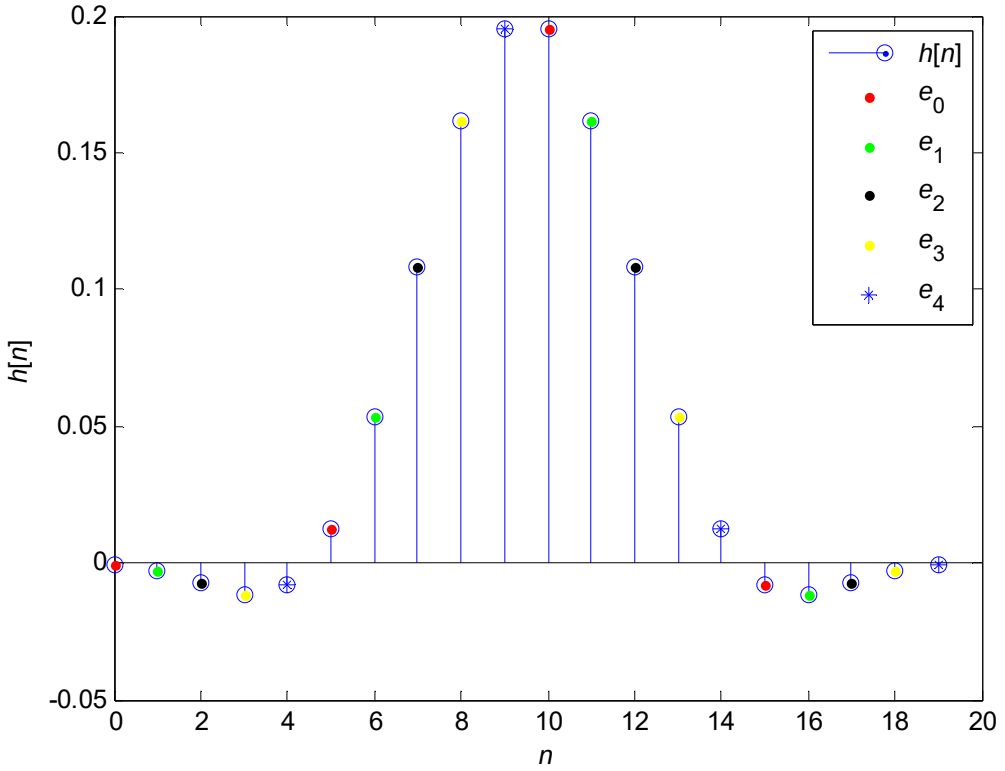
Cilj je da se računske operacije „spuste“ na nižu frekvenciju obrade

Polifazna dekompozicija - decimator



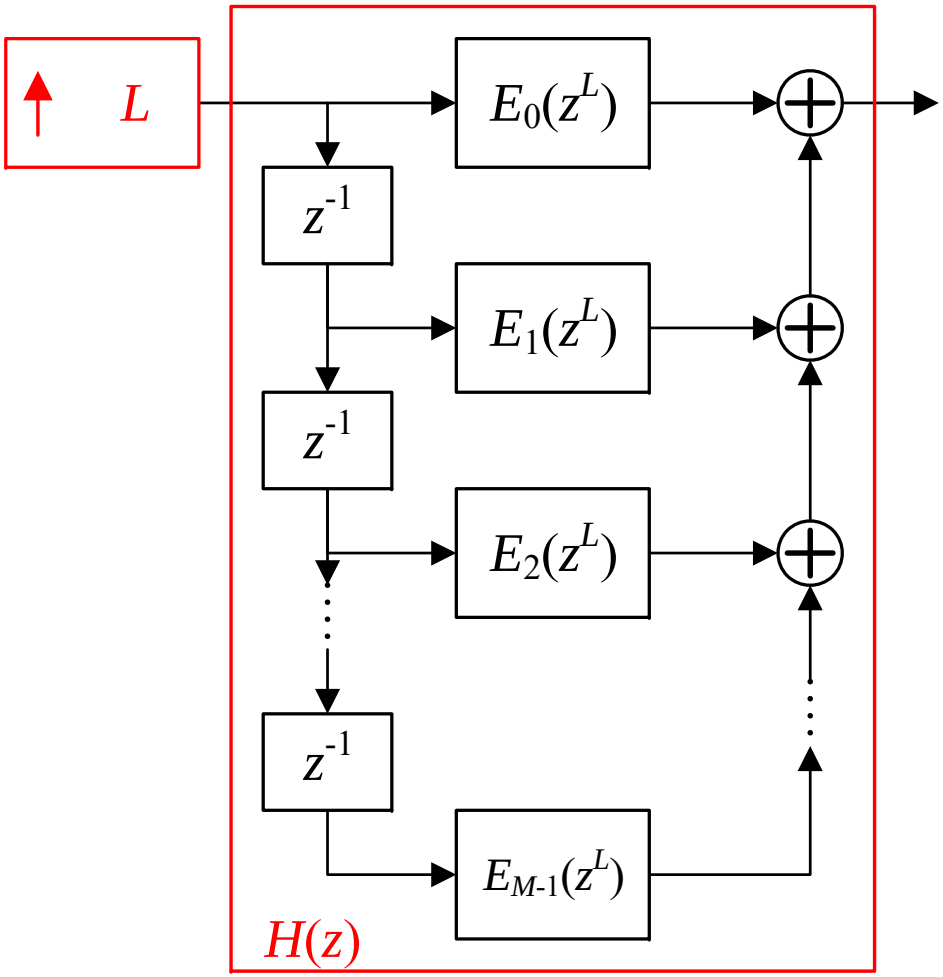
Polifazna dekompozicija - decimator

Primer M=5



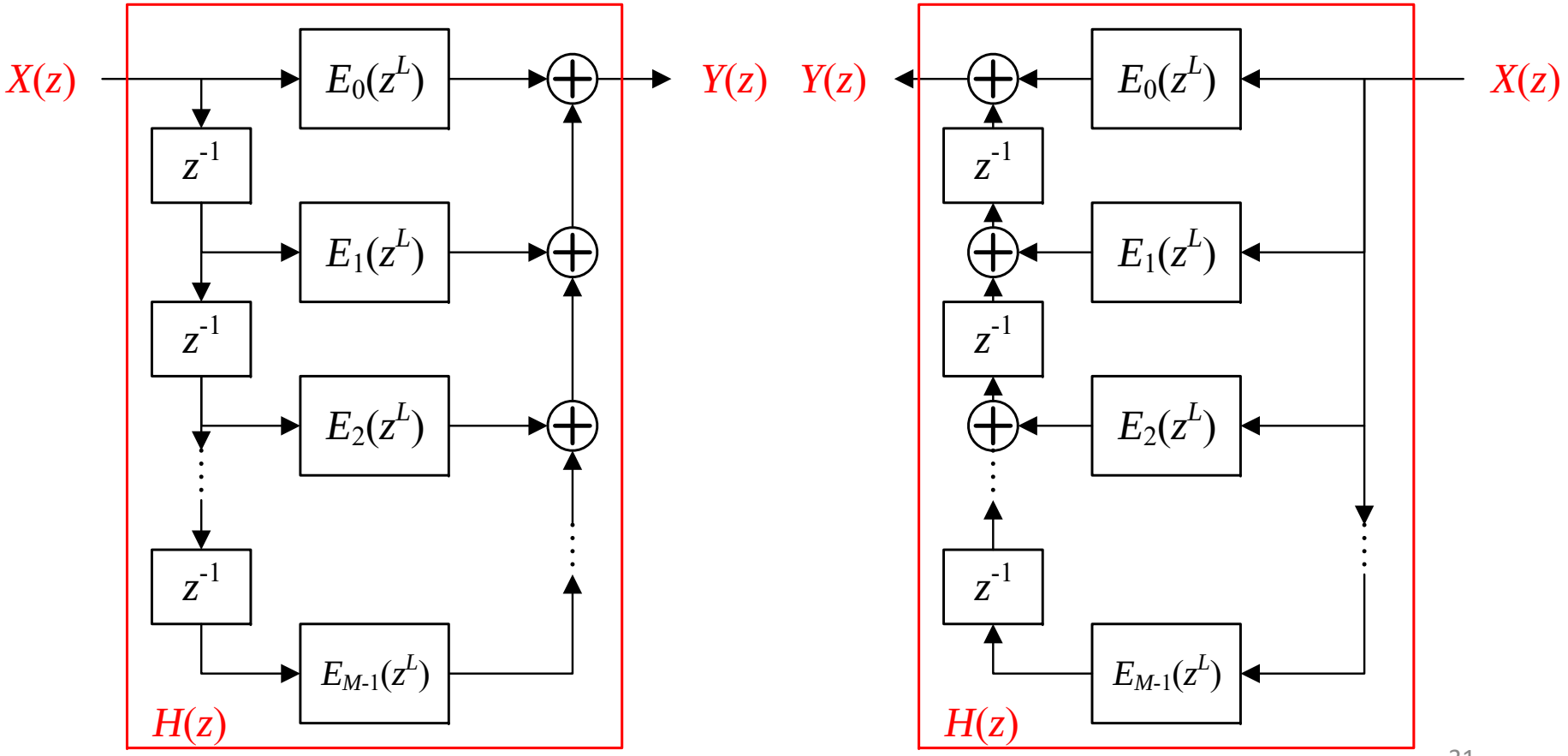
-0.0008	0.0126	0.1949	-0.0076
-0.0032	0.0532	0.1617	-0.0114
-0.0075	0.1081	0.1081	-0.0075
-0.0114	0.1617	0.0532	-0.0032
-0.0076	0.1949	0.0126	-0.0008

Polifazna dekompozicija - interpolator

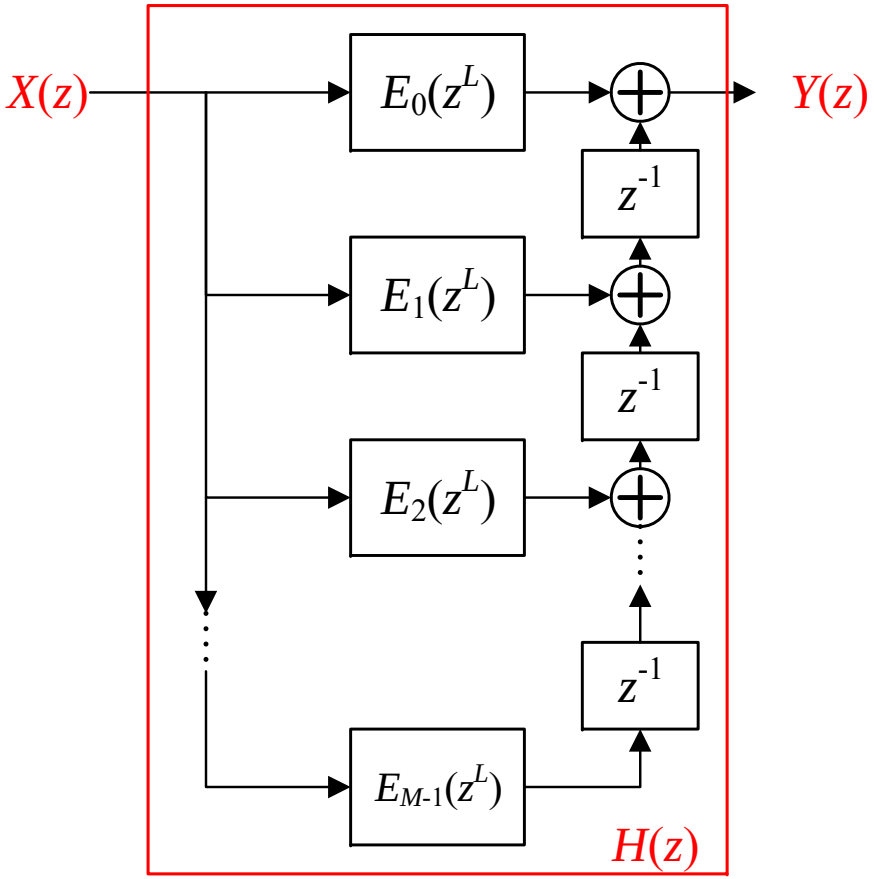


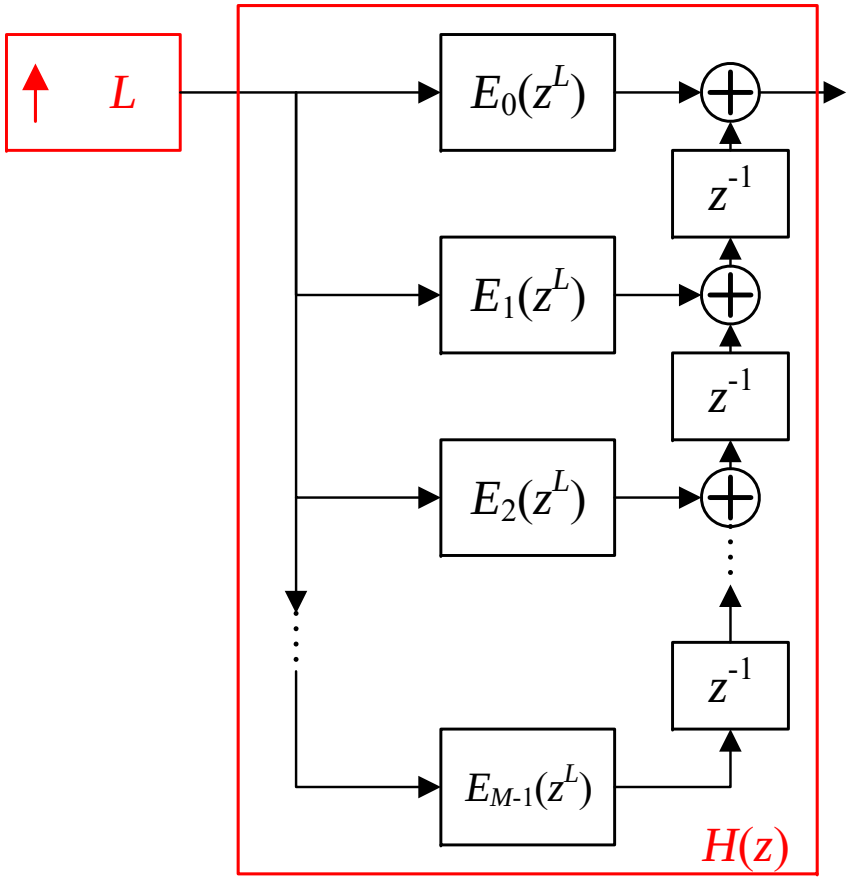
Cilj je da se računske operacije „spuste“ na nižu frekvenciju obrade, što sa ovakvom strukturom nije moguće

Transponovana struktura (podsetnik)

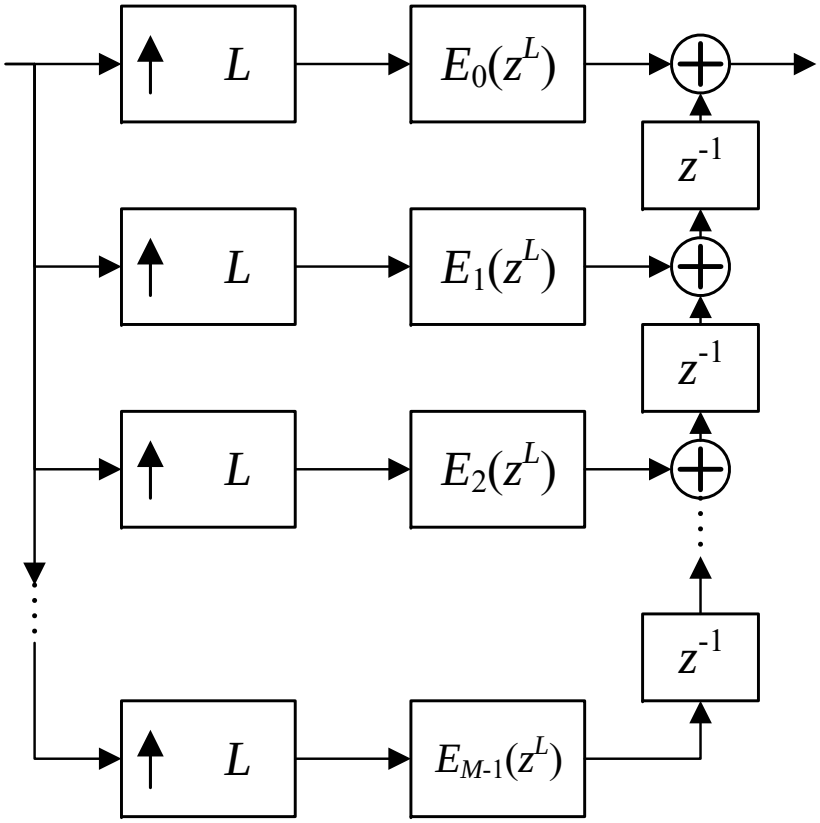


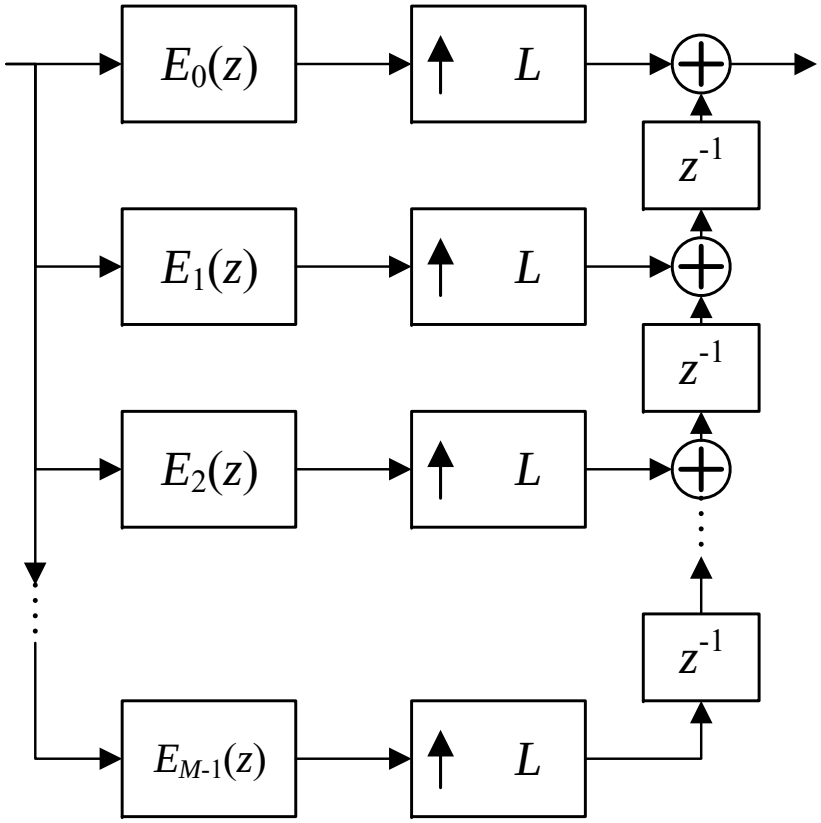
Transponovana struktura i interpolator





Transponovana struktura i interpolator





Polifazna dekompozicija - značaj

- Svaki FIR filtar može da se realizuje preko polifazne strukture
- Realizacija preko polifazne strukture omogućava da se decimatori i interpolatori realizuju tako da se računske operacije „spuste“ na niže frekvencije obrade

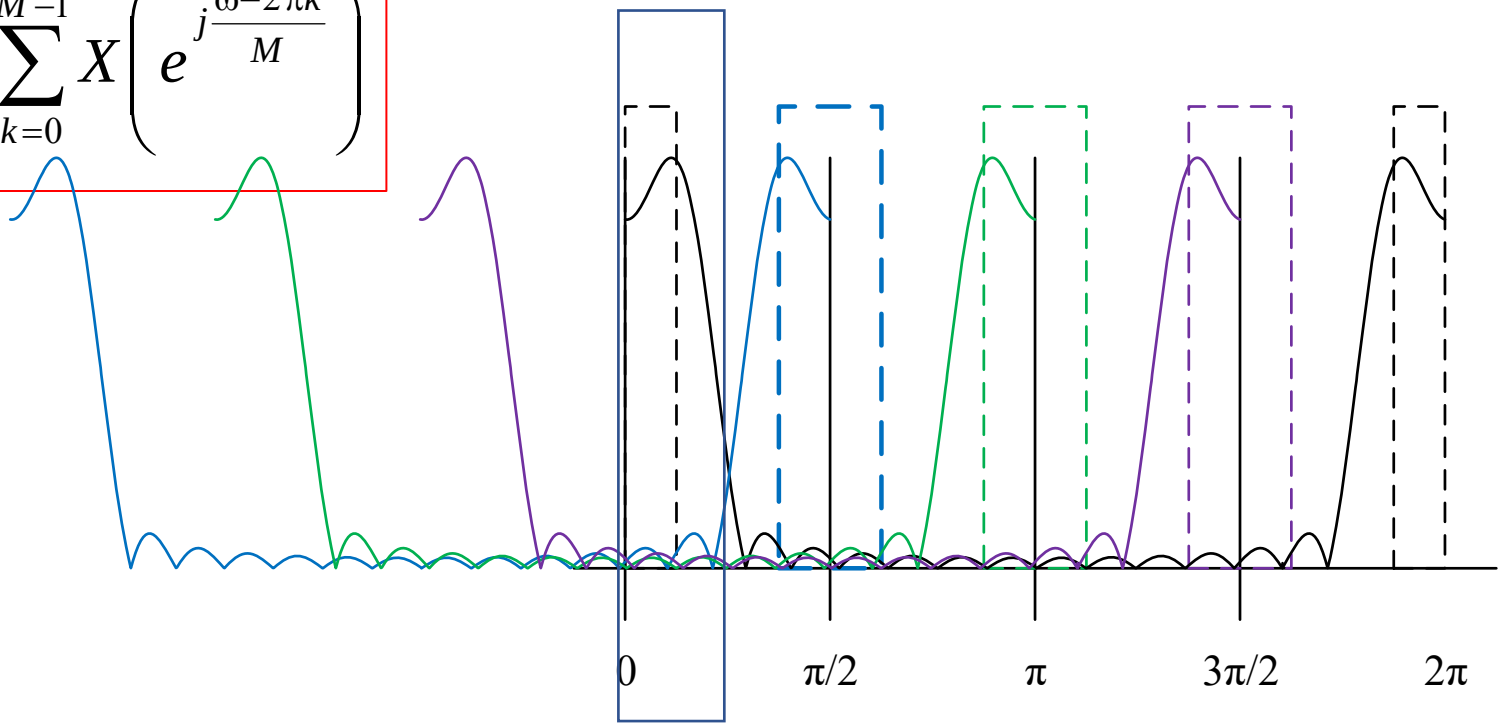
Filtri – zadavanje specifikacija - decimacija

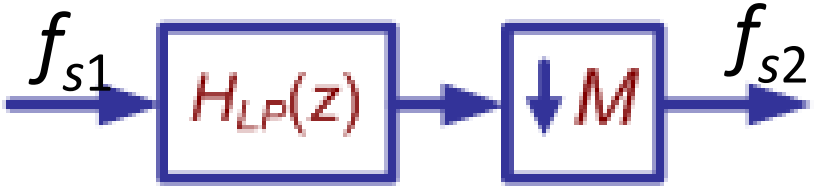
Downsamper

$$Y\left(e^{j\omega}\right)=\frac{1}{M}\sum_{k=0}^{M-1}X\left(e^{j\frac{\omega-2\pi k}{M}}\right)$$

M=4

$2\pi \leftrightarrow f_{s1}$





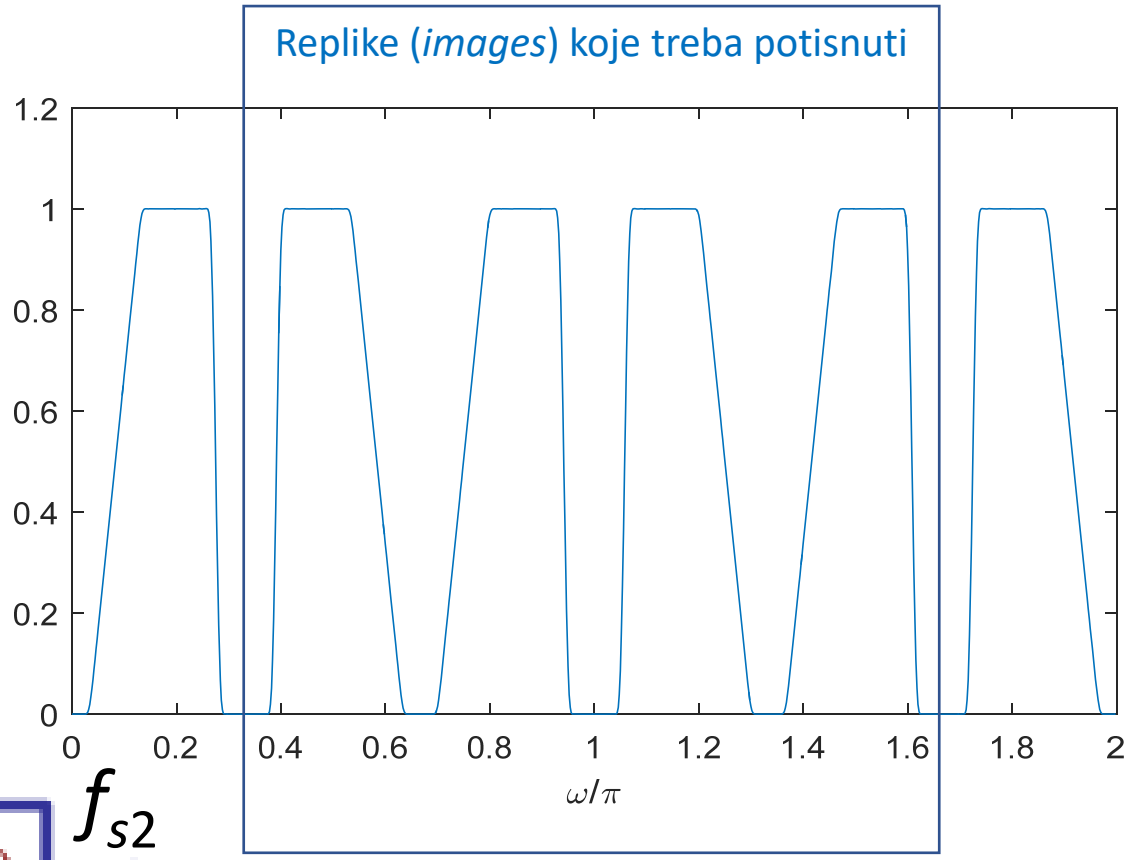
Filtri – zadavanje specifikacija - interpolacija

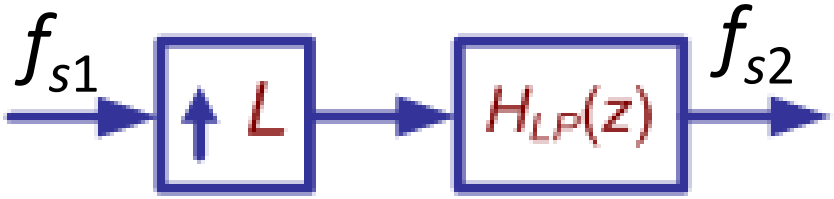
$2\pi \leftrightarrow f_{s2}$

L=3

Upsampler

$$Y\left(e^{j\omega}\right) = X\left(e^{j\omega L}\right)$$





Filtri – zadavanje specifikacija

- Specifikacije za filtar se zadaju na „višoj“ frekvenciji odabiranja (za decimator f_{s1}, za interpolator f_{s2})
- Filtri za decimaciju i interpolaciju se, u principu, projektuju na isti način

Filtri – projektovanje

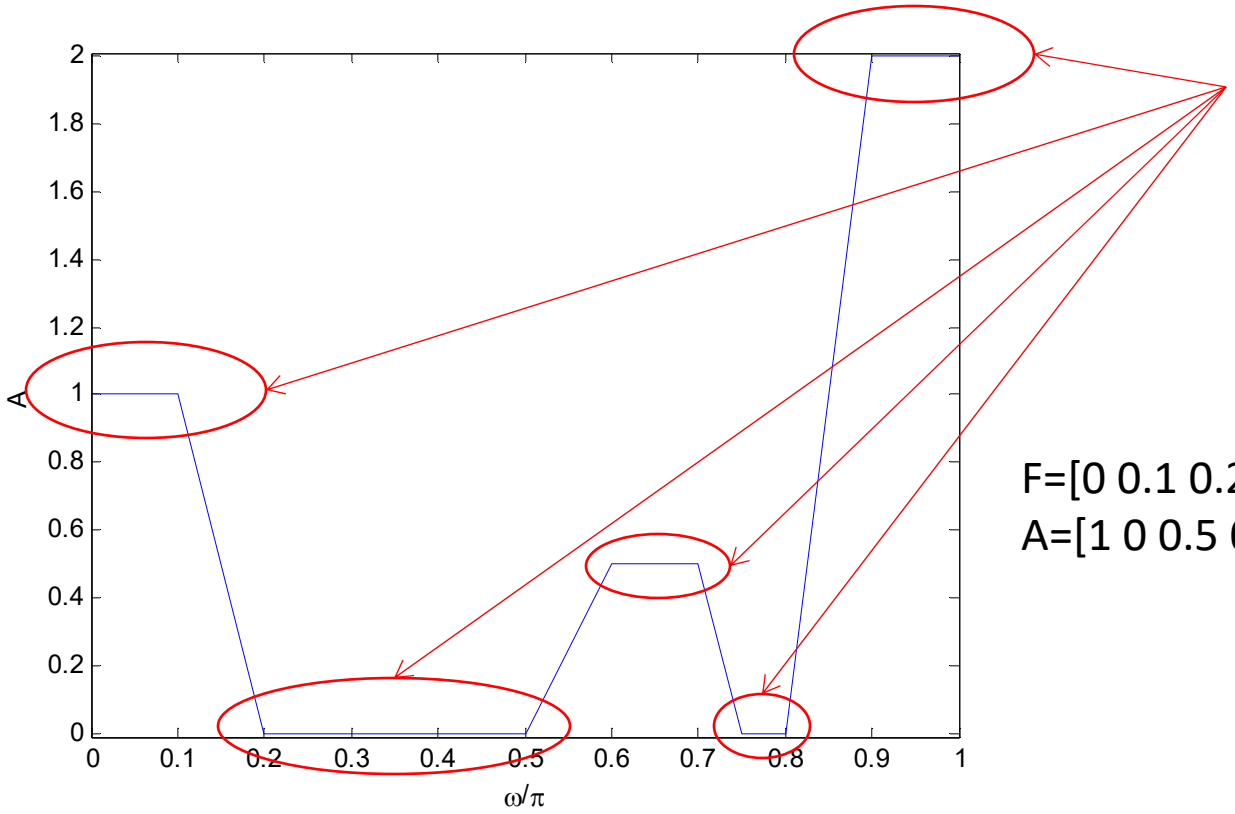
Za projetovanje FIR decimacionih/interpolacionih filtara koristimo optimalnu metodu

U MATLAB-u je to funkcija firpm (i firpmord)

[N,f,a,w]=firpmord(F,A,dev)
Funkcija podržava projektovanje FIR filtara linearne faze sa više “zadatih” opsega, svaki opseg od interesa je definisan sa dve vrednosti u vektoru frekvencija, i po jednom vrednošću u vektorima amplitude i dozvoljene devijacije (za vektor frekvencija podrazumeva se da je prva vrednost 0 a poslednja 1 i one se ne upisuju u vektor, vektor frekvencija je normalizovan sa π)
h=firpm(N,f,a,w);

Filtri – projektovanje

Primer za opšti slučaj korišćenja funcije firpmord



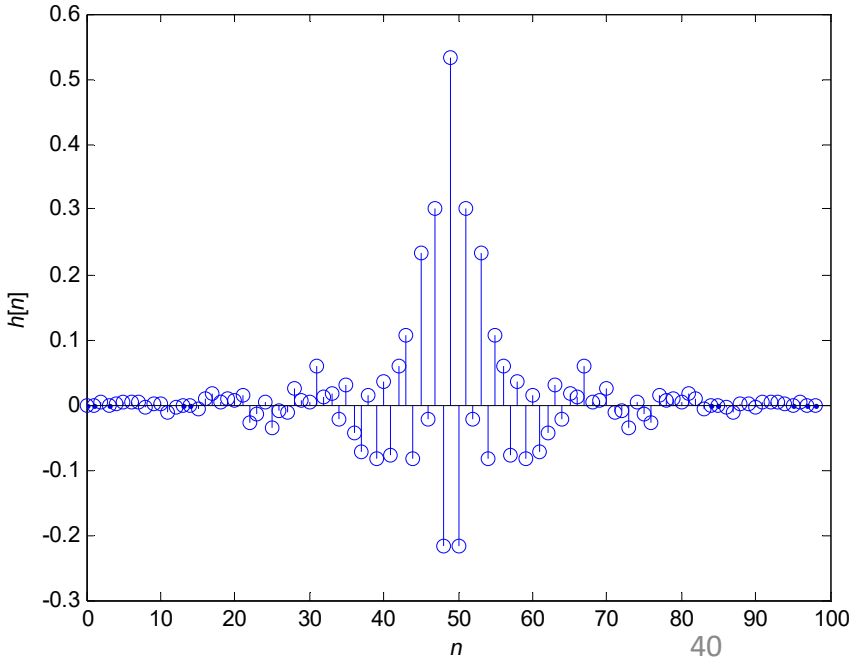
Zadati opsezi
(između su “don´t care” regije)

F=[0 0.1 0.2 0.5 0.6 0.7 0.75 0.8 0.9 1];
A=[1 0 0.5 0 2];

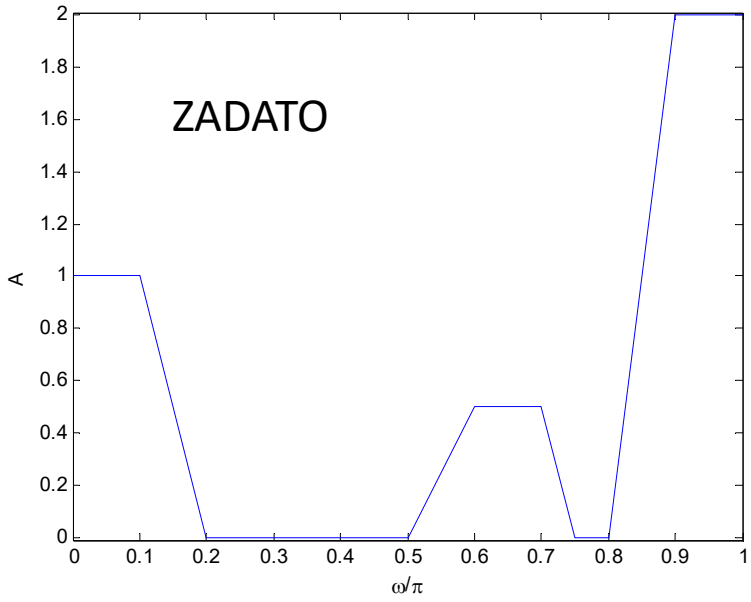
Filtri – projektovanje

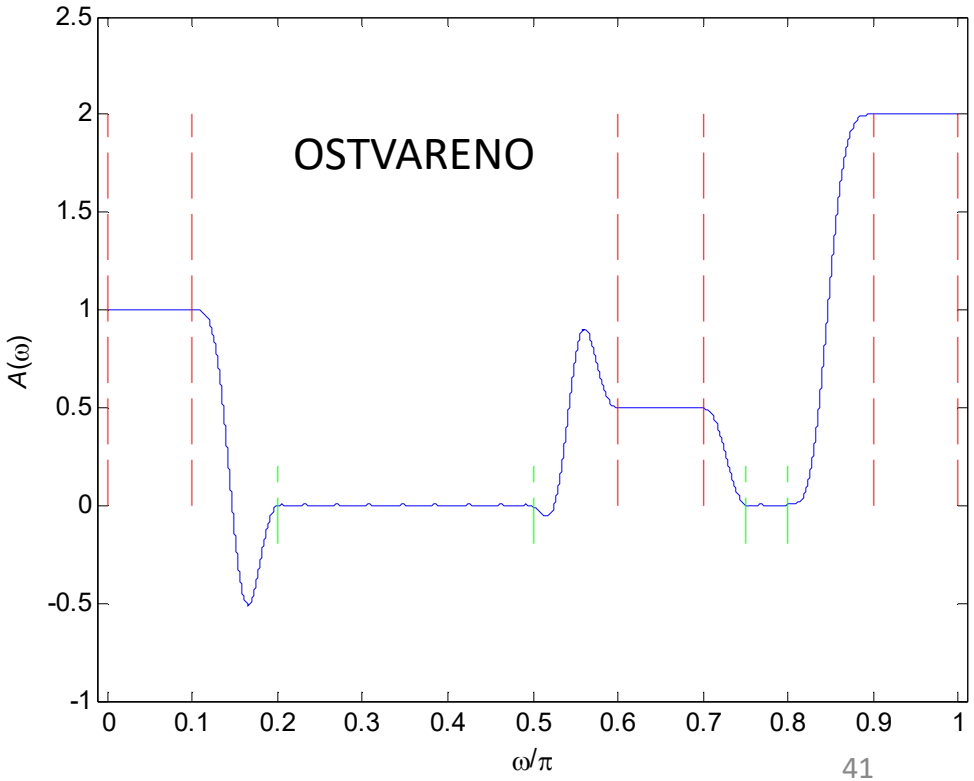
Primer za opšti slučaj korišćenja funcije firpmord

```
F=[0 0.1 0.2 0.5 0.6 0.7 0.75 0.8 0.9 1];
A=[1 0 0.5 0 2];
AA=[A;A];
figure,plot(F,AA(:));
xlabel('\omega/\pi');
ylabel('A');
ylim([-0.01 2.01]);
dev=[0.001 0.01 0.001 0.01 0.0001];
[N,f,a,d]=firpmord(F(2:end-1),A,dev);
h=firpm(N,f,a,d);
figure,stem(0:N,h);
xlabel('\itn'); ylabel('{\ith}[{\itn}]');
```

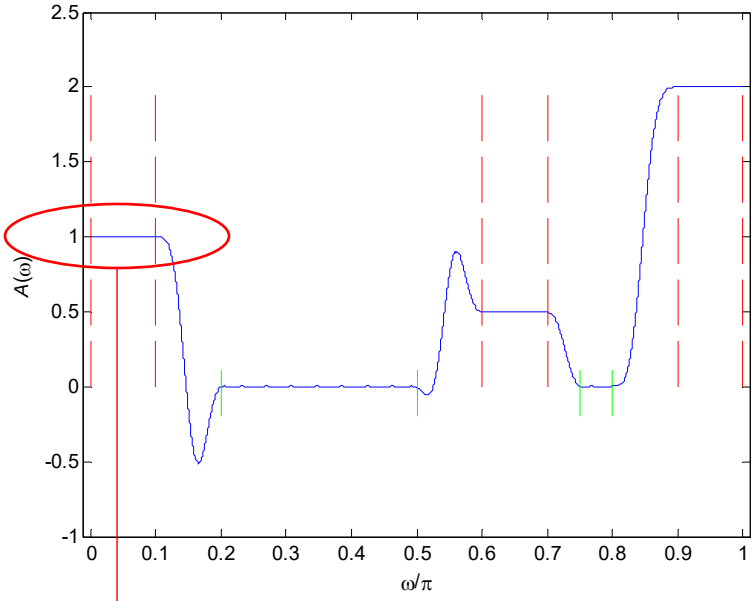


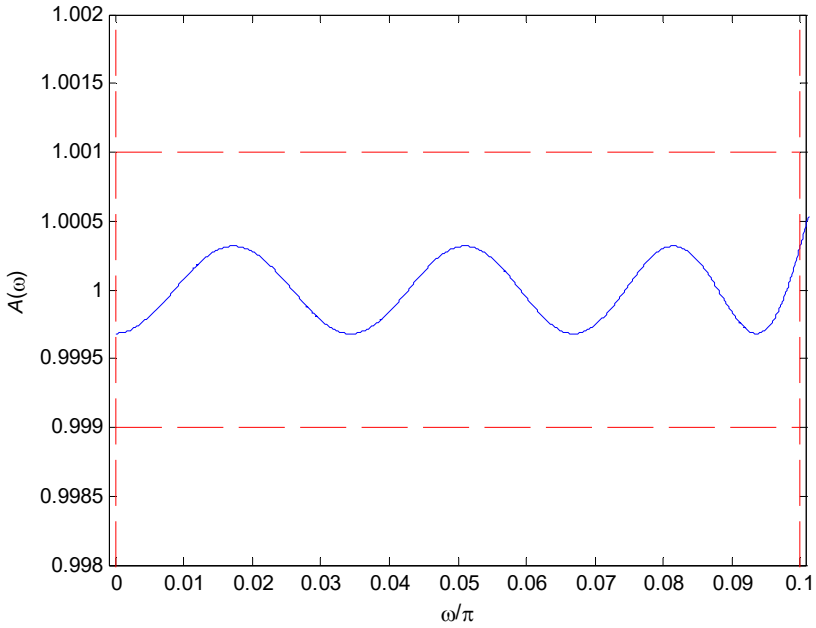
Filtri – projektovanje



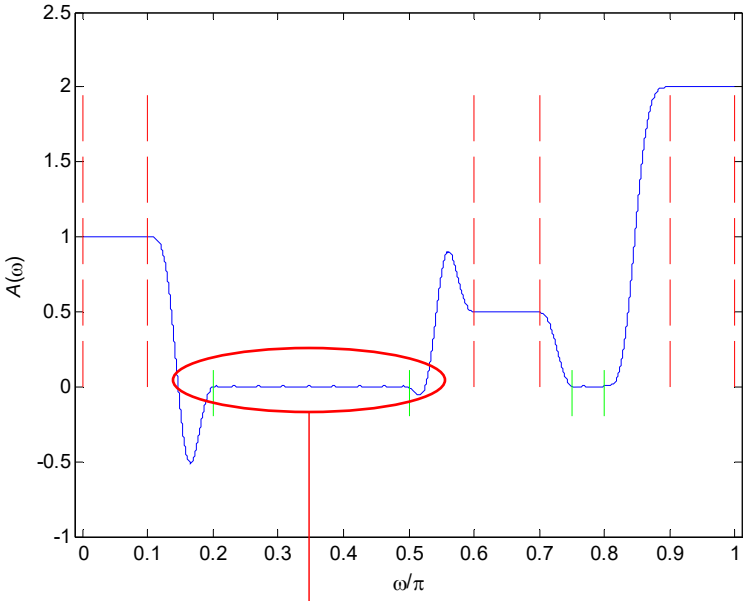


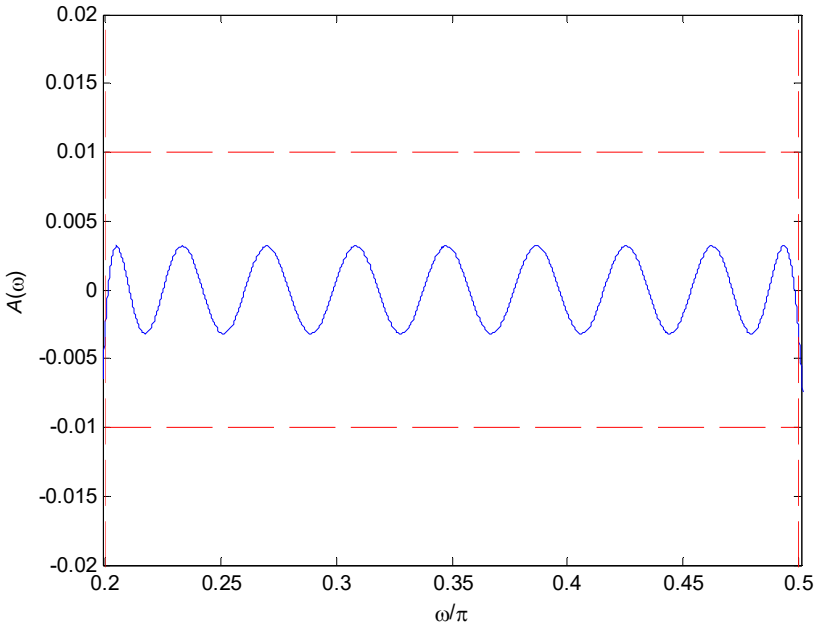
Filtri – projektovanje



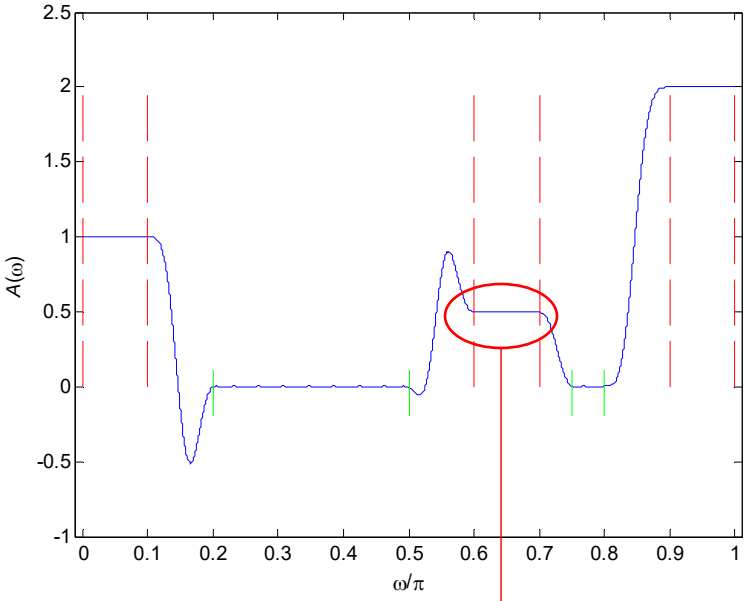


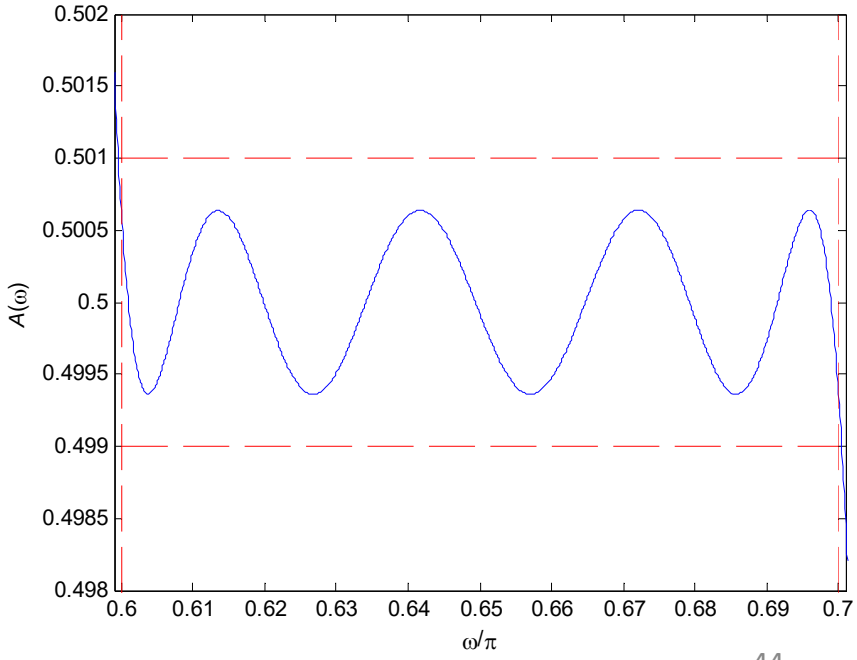
Filtri – projektovanje



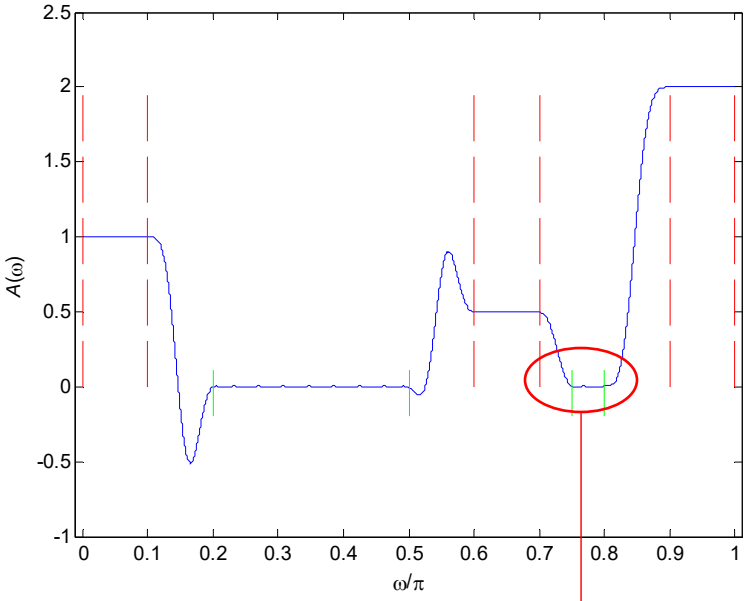


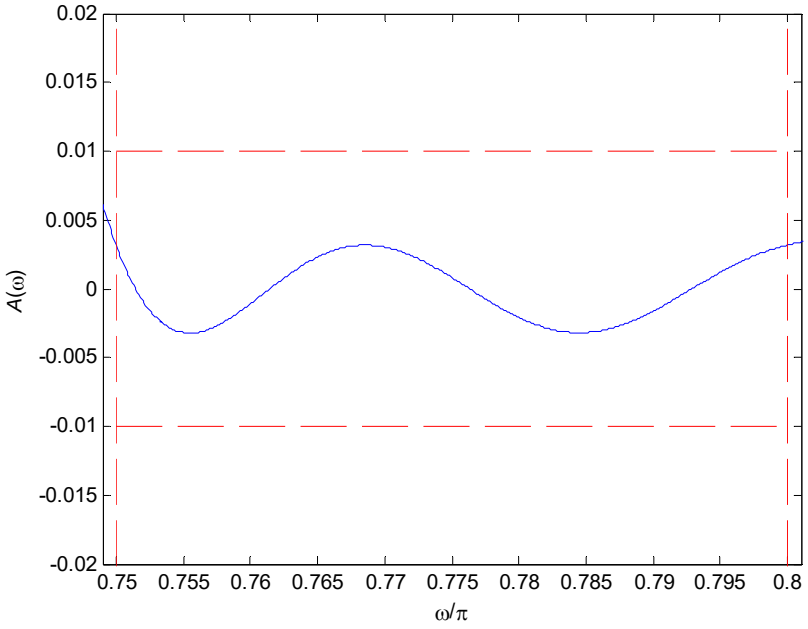
Filtri – projektovanje



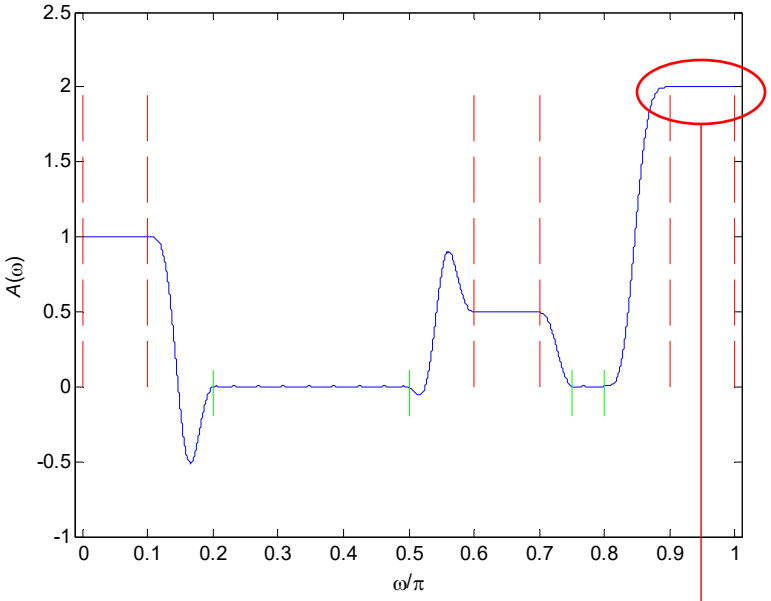


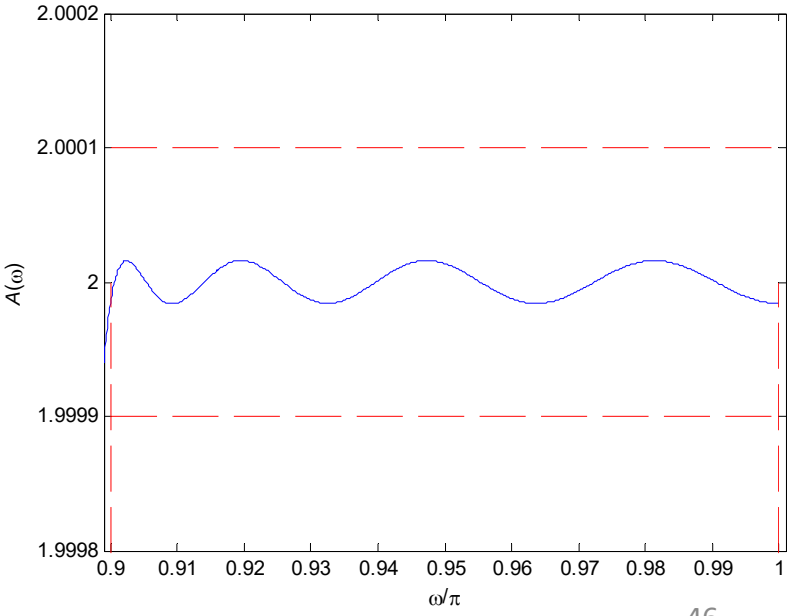
Filtri – projektovanje





Filtri – projektovanje





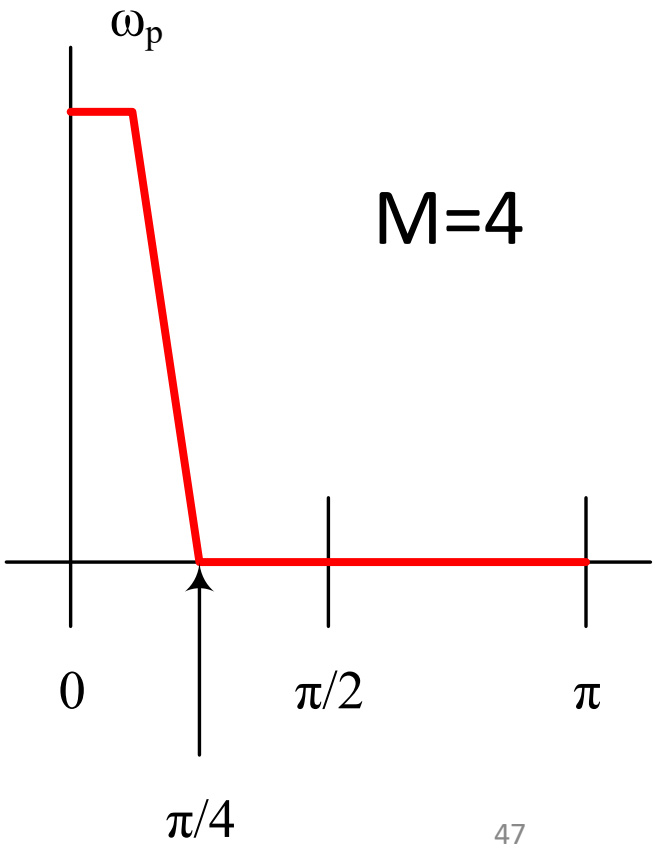
Filtri – zadavanje specifikacija FIR (1)

Propusni opseg

$$1-\delta_p \le \left|H\left(e^{j\omega}\right)\right| \le 1+\delta_p, \quad 0 \le \omega \le \omega_p$$

Nepropusni opseg

$$0 \le \left|H\left(e^{j\omega}\right)\right| \le \delta_s, \quad \frac{\pi}{M} \le \omega \le \pi$$



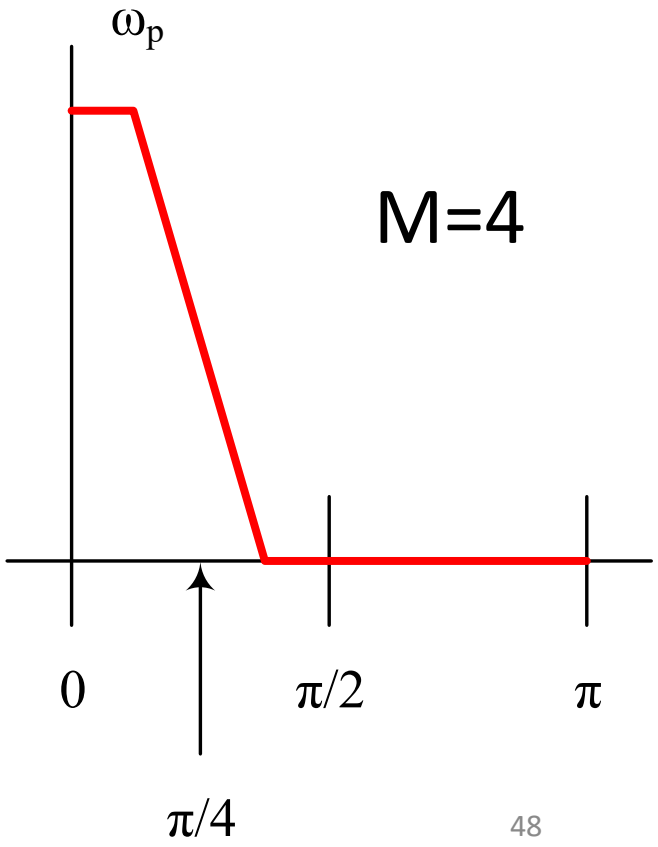
Filtri – zadavanje specifikacija FIR (2)

Propusni opseg

$$1-\delta_p \le \left|H\left(e^{j\omega}\right)\right| \le 1+\delta_p, \quad 0 \le \omega \le \omega_p$$

Nepropusni opseg

$$0 \le \left|H\left(e^{j\omega}\right)\right| \le \delta_s, \quad \frac{2\pi}{M}-\omega_p \le \omega \le \pi$$



Filtri – zadavanje specifikacija FIR (3)

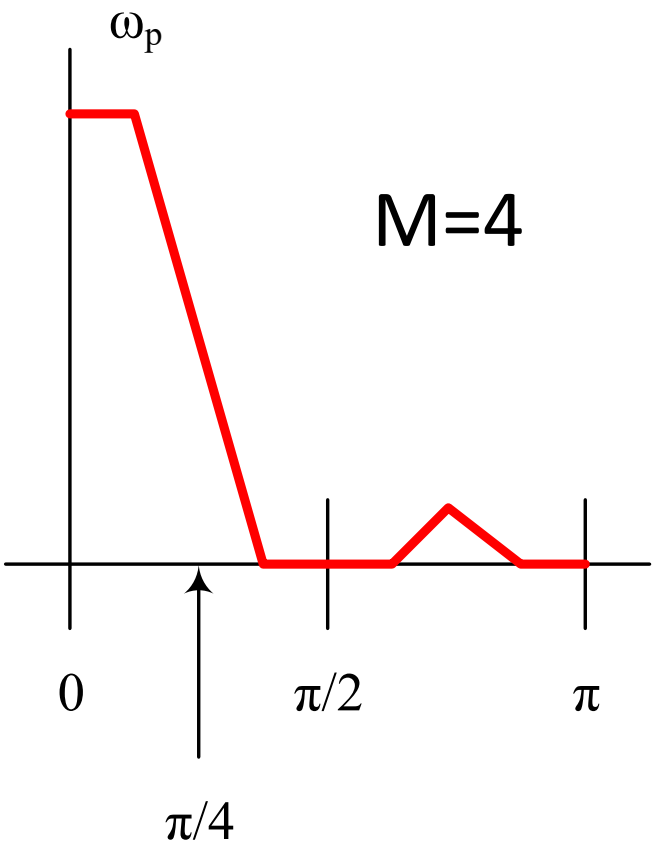
Propusni opseg

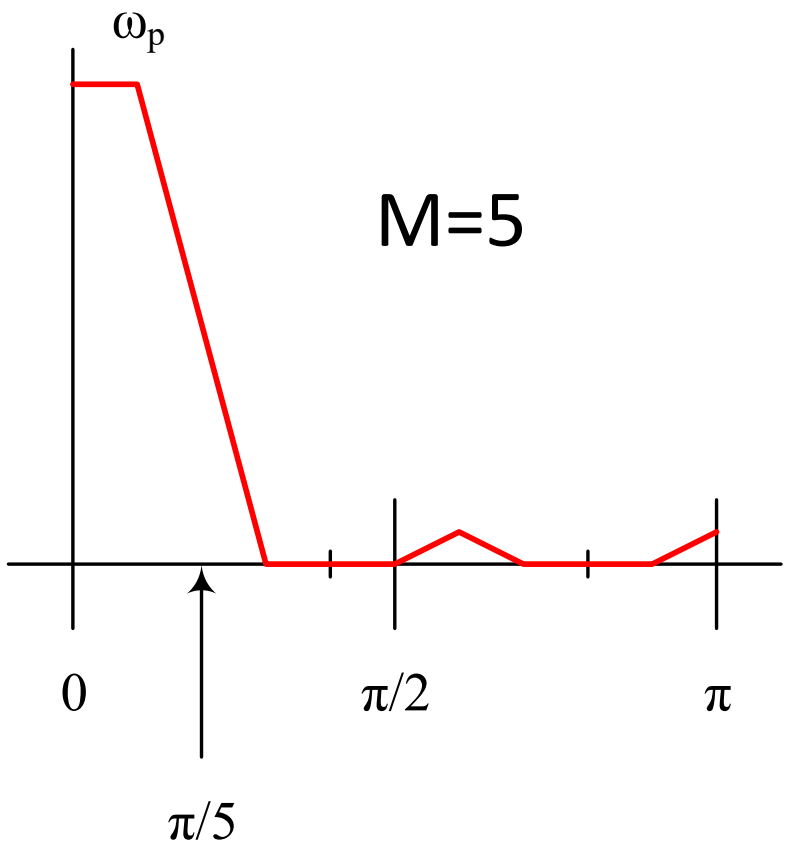
$$1-\delta_p \le \left|H\left(e^{j\omega}\right)\right| \le 1+\delta_p, \quad 0 \le \omega \le \omega_p$$

Nepropusni opseg

$$0 \le \left|H\left(e^{j\omega}\right)\right| \le \delta_s, \quad \omega \in \bigcup_{k=1}^{\left\lfloor \frac{M}{2} \right\rfloor} \left[\frac{2k\pi}{M} + \omega_p,\ \min\left\{ \frac{2k\pi}{M} - \omega_p, \pi \right\} \right]$$

Filtri – zadavanje specifikacija FIR (3)

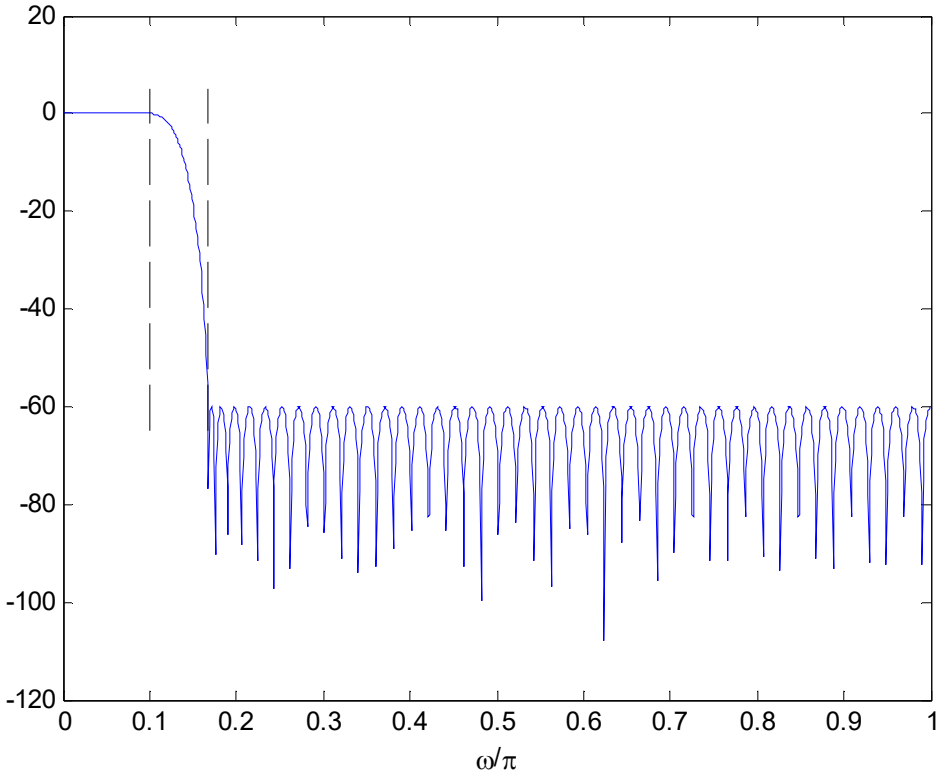




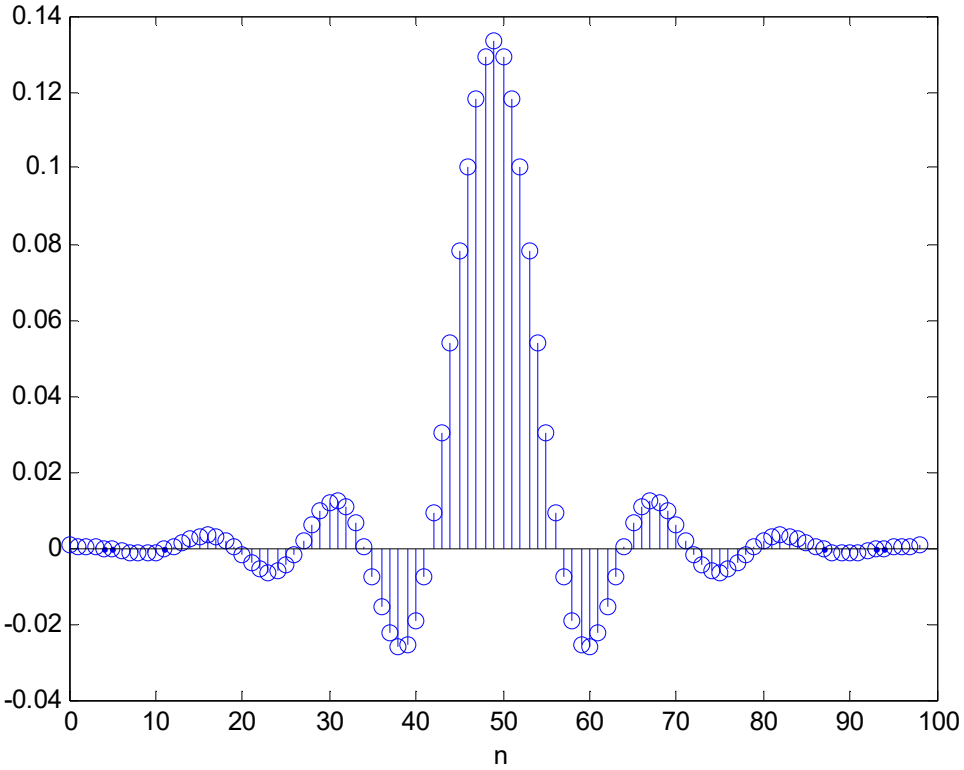
Decimator/interpolator tip 1

```
close all
clear
% tip 1
M=6;
wp=0.1; % 0.1*pi
ws1=1/M; % pi/M
dev=0.001; %deltap=deltas As=60dB
[N1,f1,a1,w01]=firpmord([wp ws1],[1 0],[dev dev])
h1=firpm(N1,f1,a1,w01);
[H1,w]=freqz(h1,1,1000);
figure,plot(w/pi,20*log10(abs(H1)));
l1=line([wp wp],[-65 5]);
l2=line([ws1 ws1],[-65 5]);
set([l1 l2],'color',[0 0 0],'linestyle','--');
xlabel('\omega/\pi');
figure,stem(0:N1,h1); xlabel('n');
figure,zplane(h1);
```

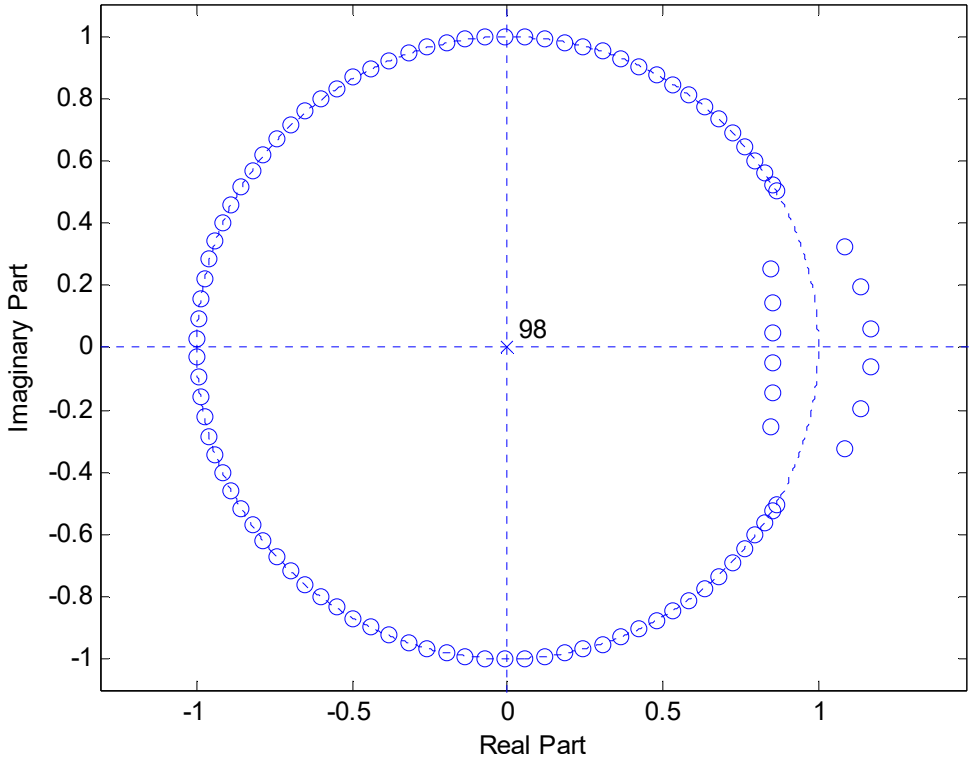
Decimator/interpolator tip 1



Decimator/interpolator tip 1



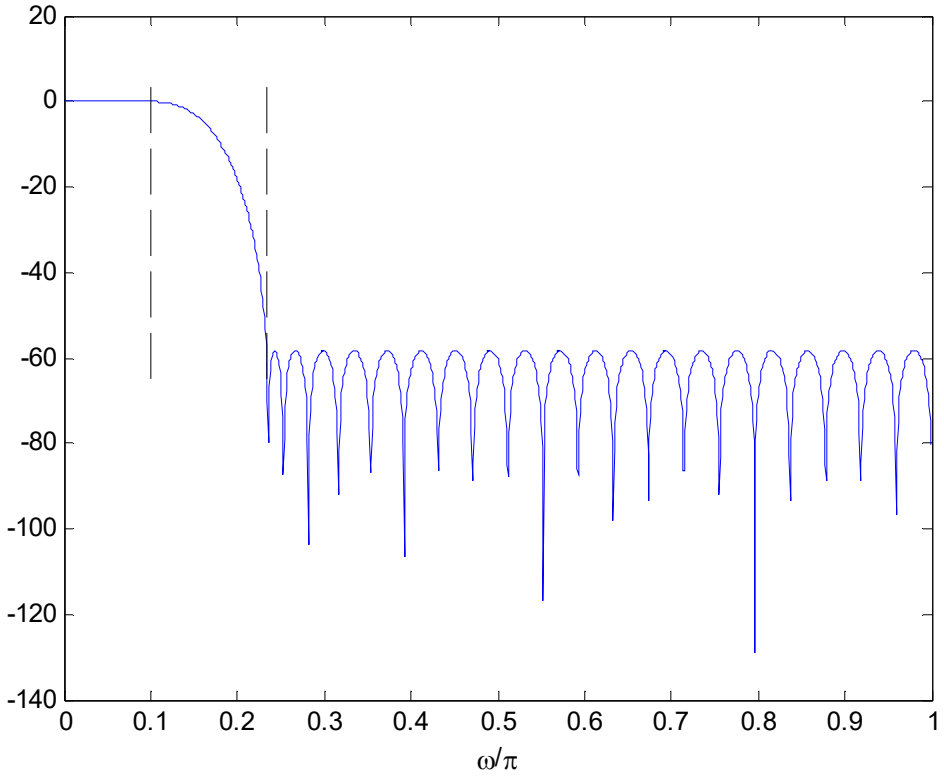
Decimator/interpolator tip 1



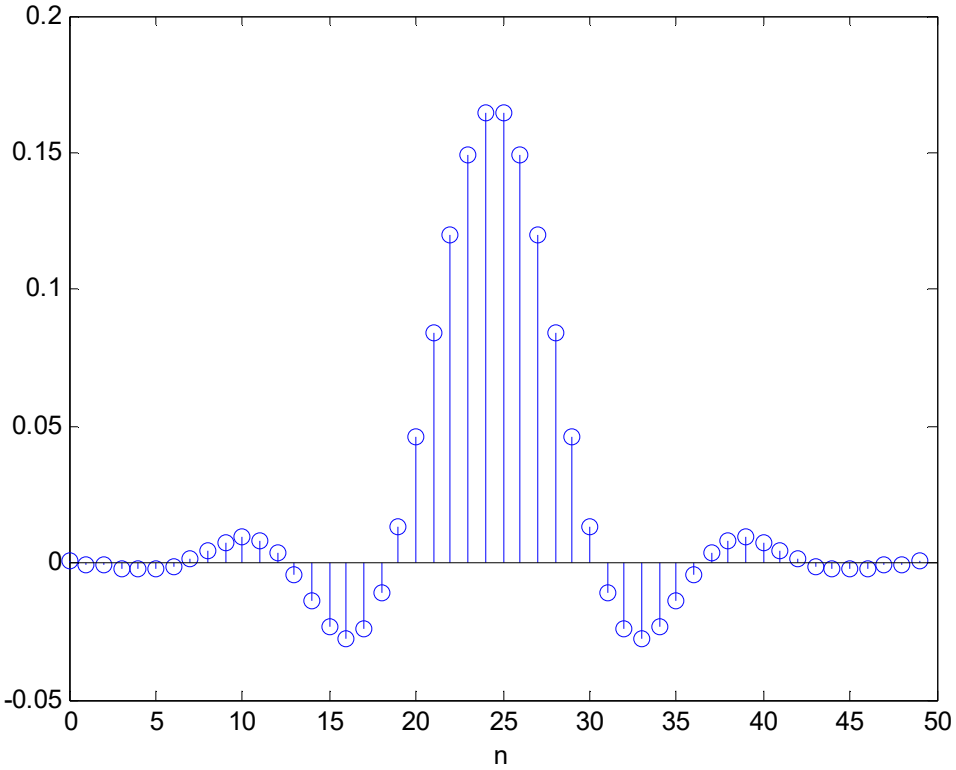
Decimator/interpolator tip 2

```
close all
clear
% tip 2
M=6;
wp=0.1; % 0.1*pi
ws2=2/M-wp; % NPO
dev=0.001; %deltap=deltas As=60dB
[N2,f2,a2,w02]=firpmord([wp ws2],[1 0],[dev dev]);
N2=N2+2;
h2=firpm(N2,f2,a2,w02);
[H2,w]=freqz(h2,1,1000);
figure,plot(w/pi,20*log10(abs(H2)));
l1=line([wp wp],[-65 5]);
l2=line([ws2 ws2],[-65 5]);
set([l1 l2],'color',[0 0 0],'linestyle','--');
xlabel('\omega/\pi');
figure,stem(0:N2,h2); xlabel('n')
figure, zplane(h2,1);
```

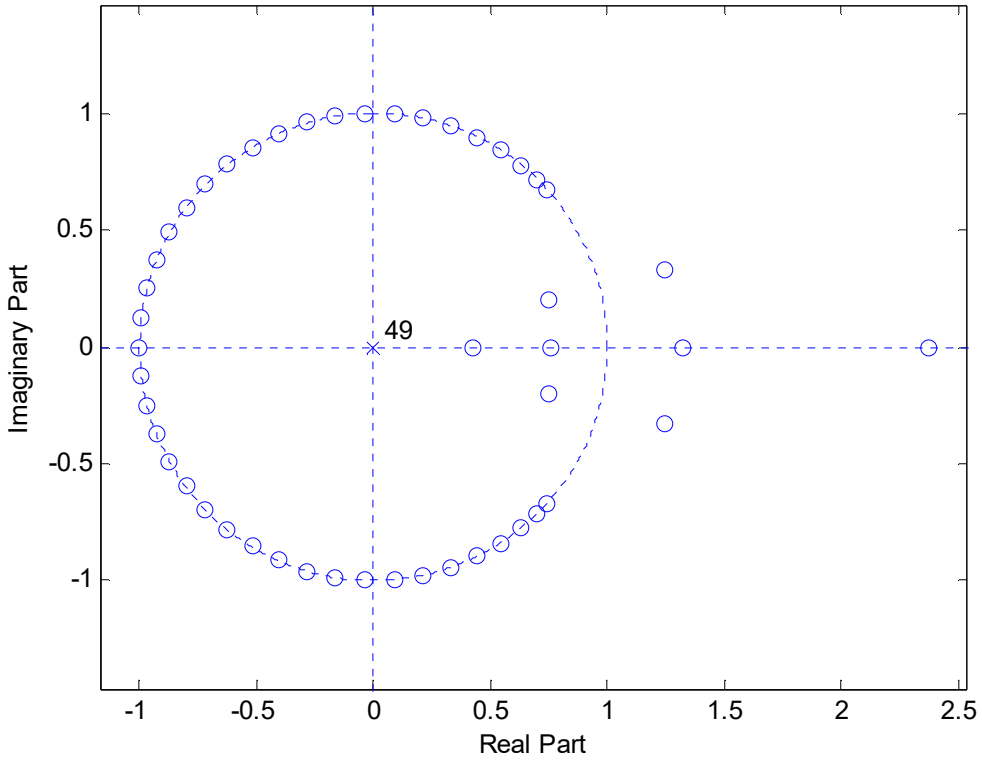
Decimator/interpolator tip 2



Decimator/interpolator tip 2



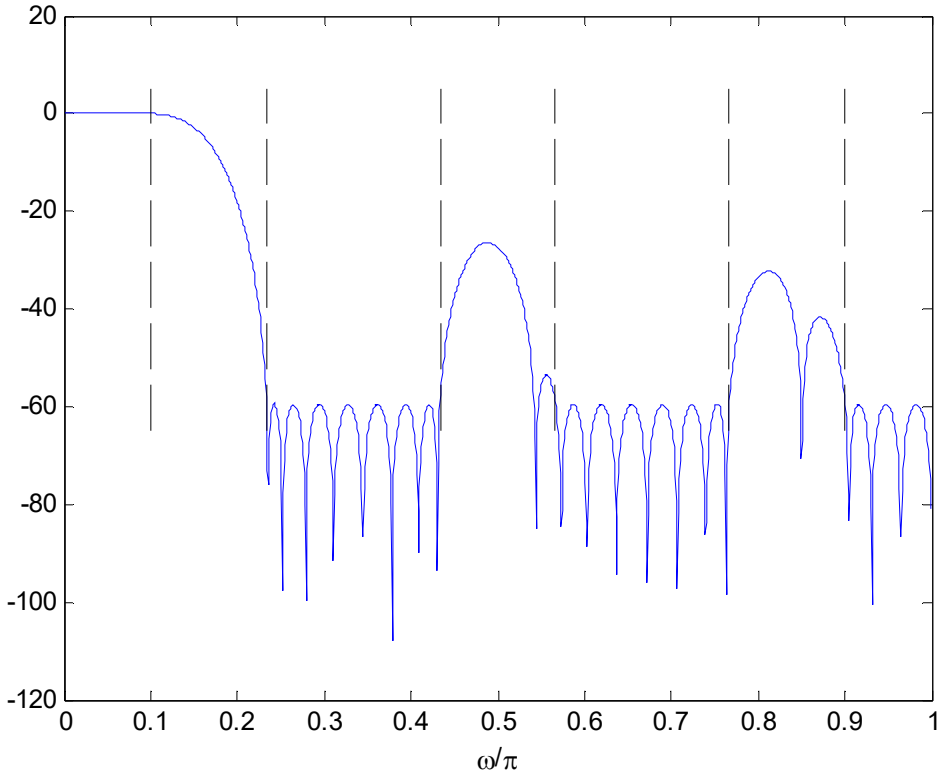
Decimator/interpolator tip 2



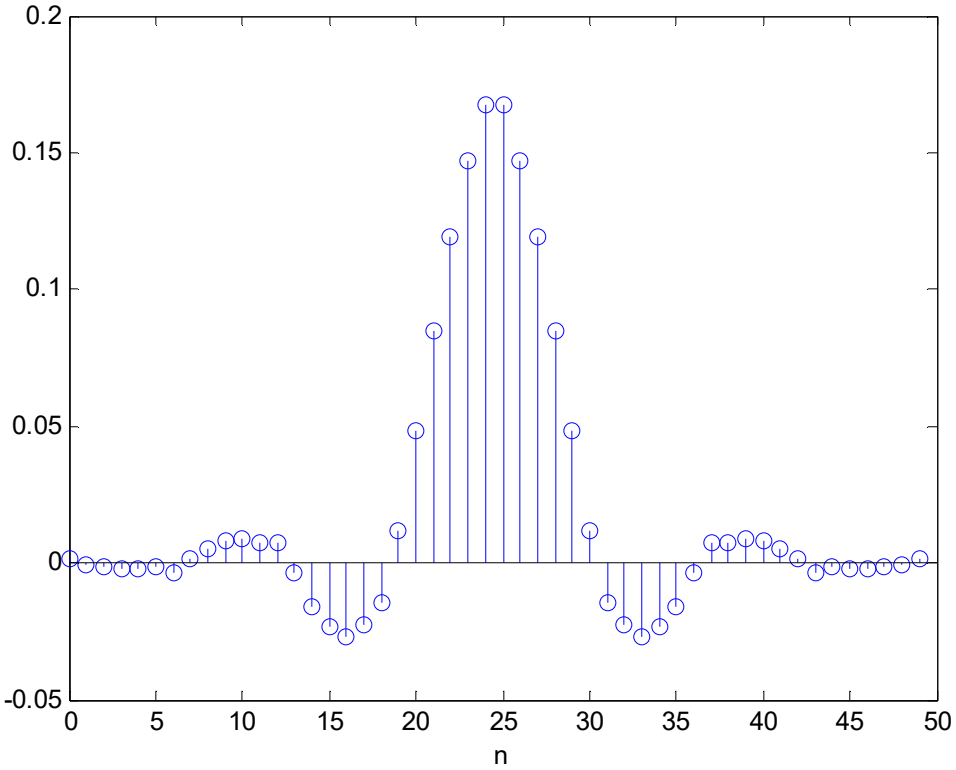
Decimator/interpolator tip 3

```
close all
clear
% tip 3
M=6;
wp=0.1; % 0.1*pi
ws3=[2/M-wp 2/M+wp 4/M-wp 4/M+wp 6/M-wp] % 3 nepropusna, treci do pi
dev=0.001; %deltap=deltas As=60dB
[N3,f3,a3,w03]=firpmord([wp ws3],[1 0 0 0],[dev dev dev dev])
h3=firpm(N3,f3,a3,w03);
[H3,w]=freqz(h3,1,1000);
figure,plot(w/pi,20*log10(abs(H3)));
xlabel('\omega/\pi');
figure,stem(0:N3,h3), xlabel('n');
figure, zplane(h3,1);
```

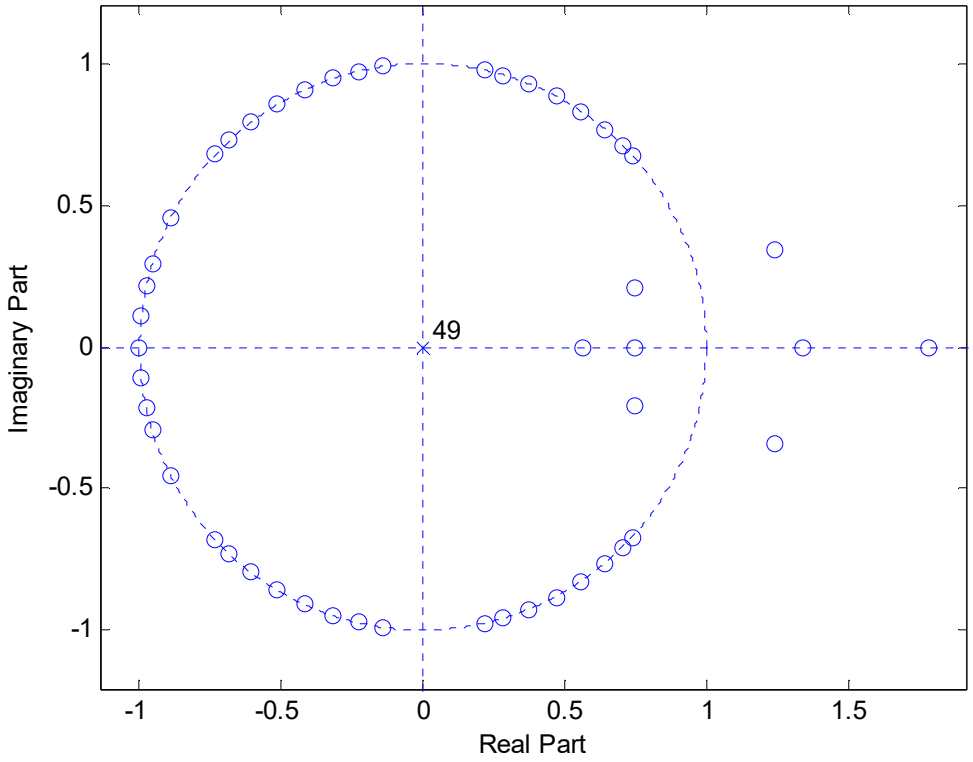
Decimator/interpolator tip 3



Decimator/interpolator tip 3



Decimator/interpolator tip 3



Filtri – zadavanje specifikacija, tipovi 1, 2 i 3