



BENEFIT

Boosting the telecommunications
engineer profile to meet modern
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Sistemi sa više brzina (13M031SVB)

Deo 6

Master studije

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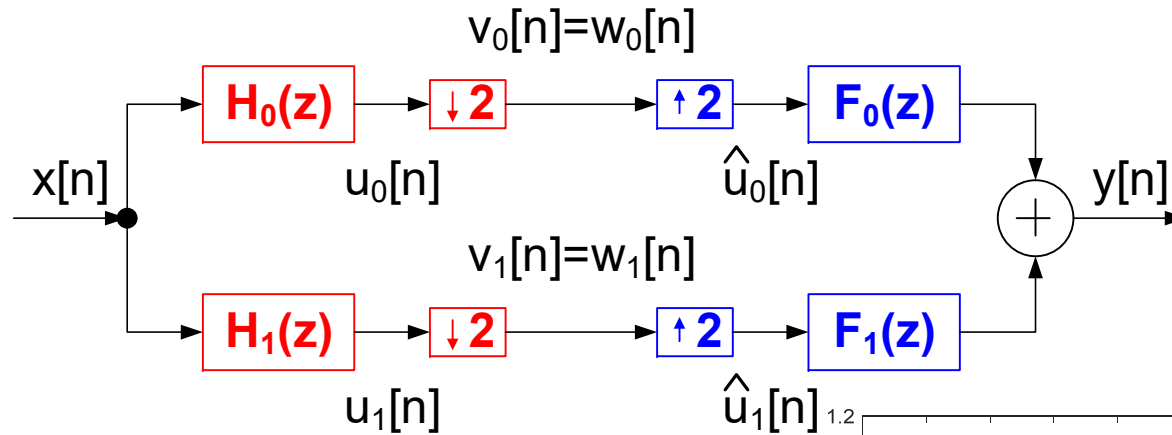
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Digitalne banke filtara - realizacije

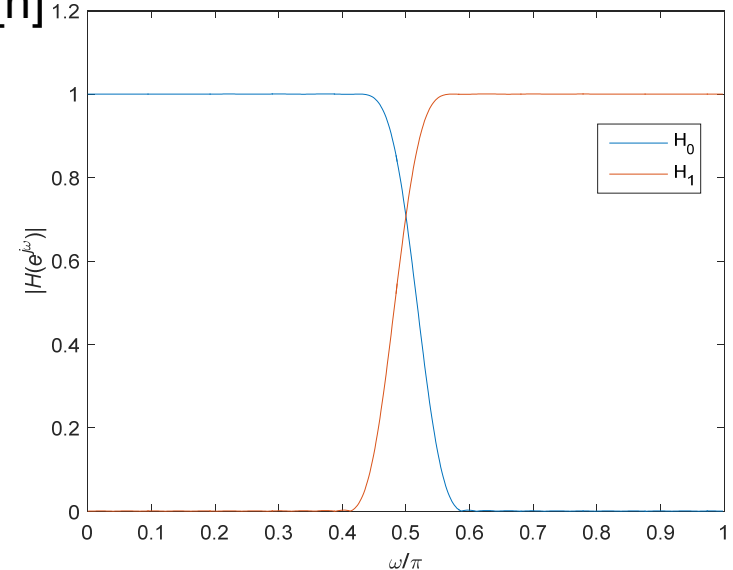
- Minimizacija broja računskih operacija
- Računske operacije na nižoj frekvenciji odabiranja (ako je *multirate* sistem)
- Minimizacija efekata konačne dužine kodne reči

Primer – dvokanalna QMF banka



$$H_1(z) = H_0(-z)$$

$$F_0(z) = 2H_1(-z), \quad F_1(z) = -2H_0(-z)$$



Primer – dvokanalna QMF banka

$$H_0(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + \dots$$

$$H_0(z) = h_0 + h_2 z^{-2} + \dots$$

$$+ z^{-1} (h_1 + h_3 z^{-2} + \dots) = E_0(z^2) + z^{-1} E_1(z^2)$$

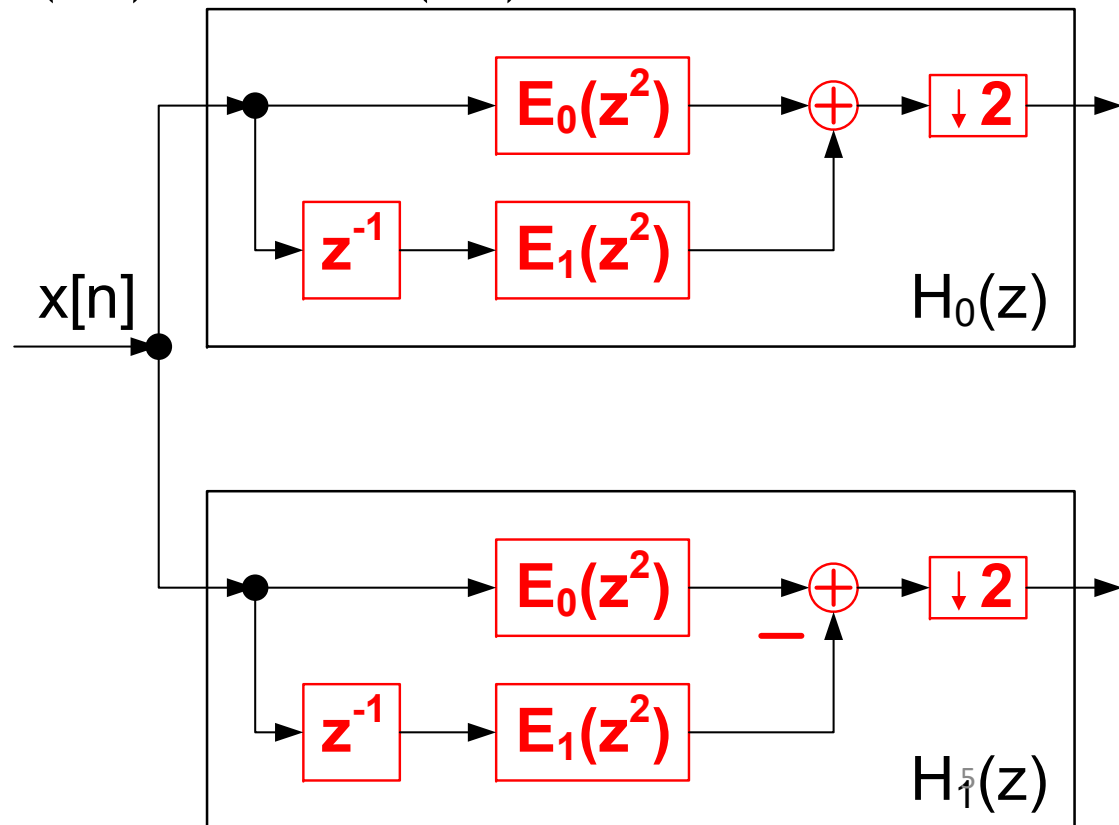


Polifazna realizacija filtra $H_0(z)$

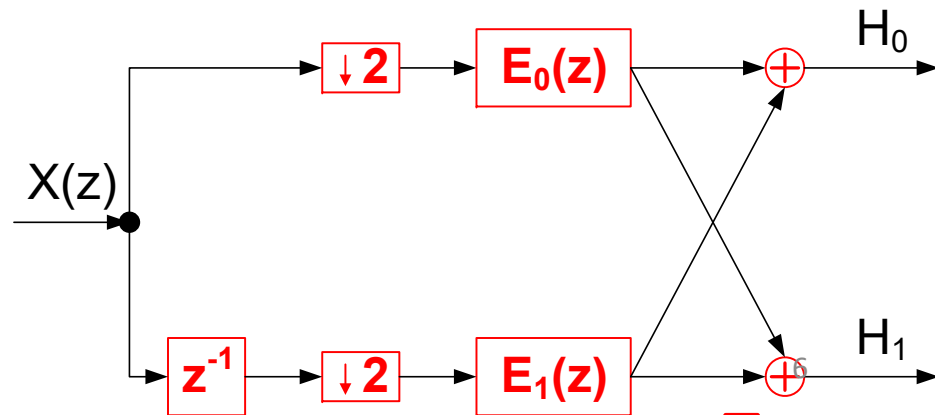
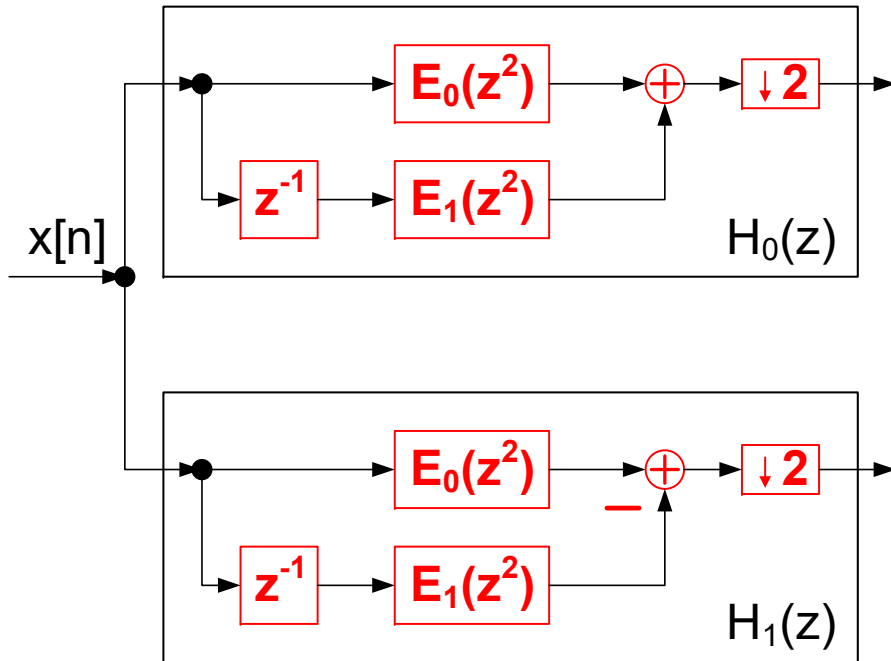
Primer – dvokanalna QMF banka

$$H_0(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

$$H_1(z) = H_0(-z) = E_0(z^2) - z^{-1}E_1(z^2)$$



Primer – dvokanalna QMF banka



Primer – dvokanalna QMF banka

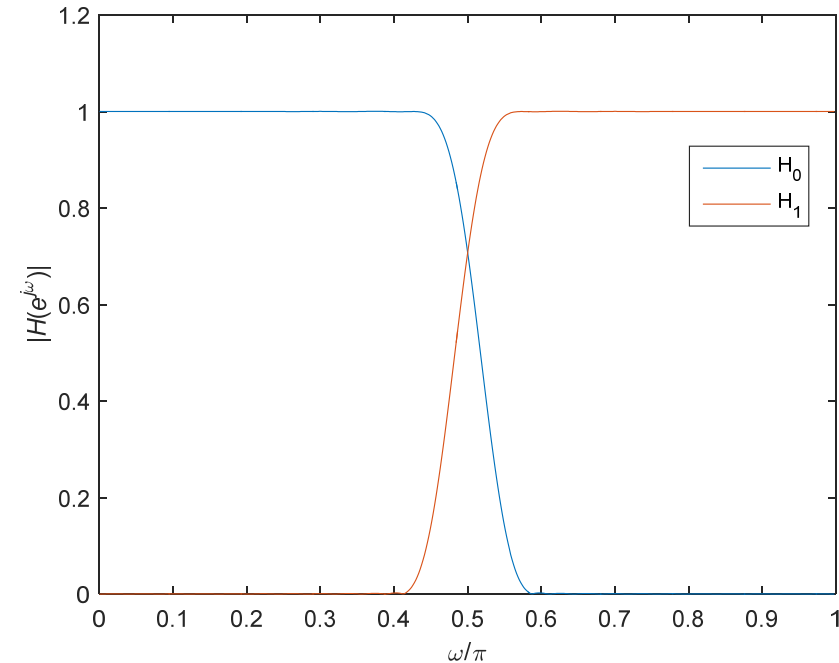
$$H_0(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

$$H_1(z) = H_0(-z) = E_0(z^2) - z^{-1}E_1(z^2)$$

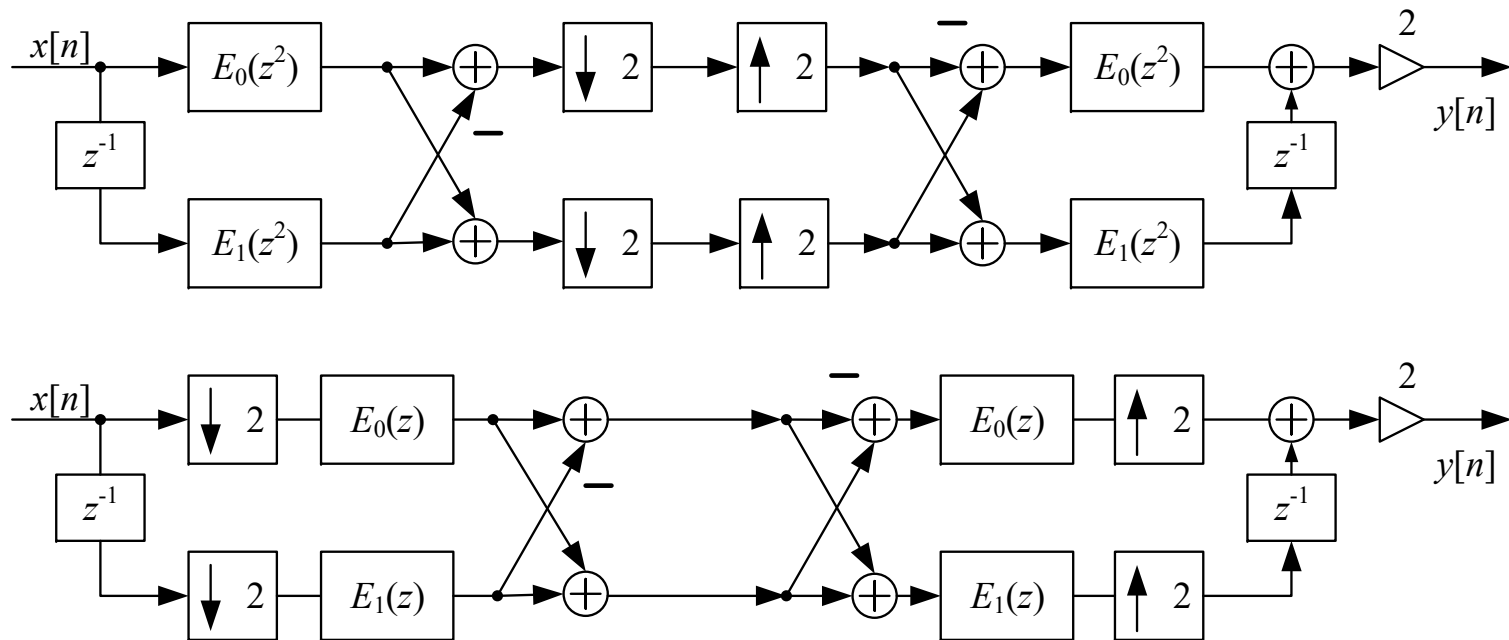
$$F_0(z) = 2H_1(-z), \quad F_1(z) = -2H_0(-z)$$

$$F_0(z) = 2(E_0(z^2) + z^{-1}E_1(z^2))$$

$$F_1(z) = -2(E_0(z^2) - z^{-1}E_1(z^2))$$



Efikasna realizacija – dvokanalna QMF banka



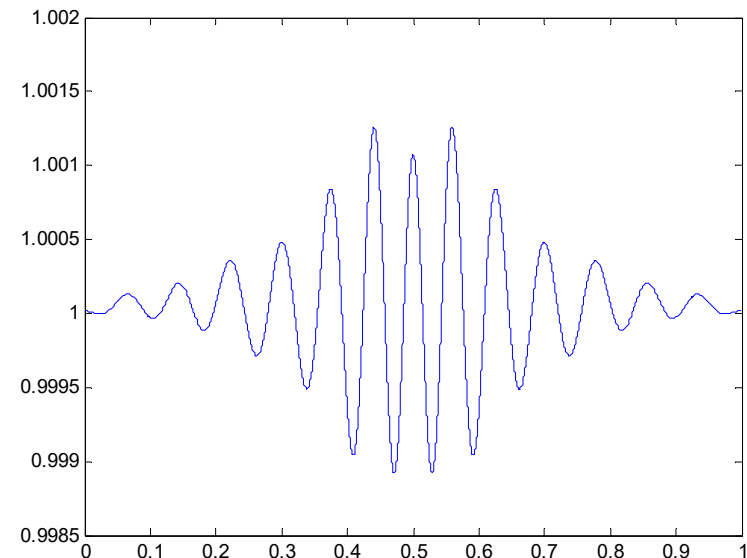
Polifazna realizacija

$$H_0(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

$$H_1(z) = E_0(z^2) - z^{-1}E_1(z^2)$$

$$\begin{aligned} T(z) &= H_0(z)H_1(-z) - H_0(-z)H_1(z) \\ &= (E_0(z^2) + z^{-1}E_1(z^2))(E_0(z^2) + z^{-1}E_1(z^2)) - \\ &\quad (E_0(z^2) - z^{-1}E_1(z^2))(E_0(z^2) - z^{-1}E_1(z^2)) = \\ &= 4z^{-1}E_0(z^2)E_1(z^2) \end{aligned}$$

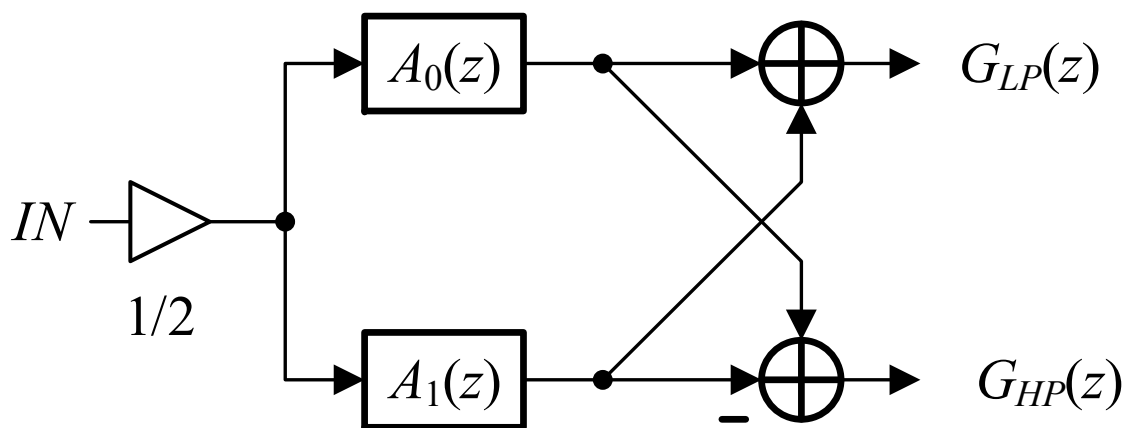
```
close all; clear;
N=47;
h0=[0.00040829340,-0.00061083240,-
0.00066471290,0.0015016570,0.00089979030,-
0.0029611340,-0.00095592250,...
0.0051489700,0.00063647700,-
0.0082474350,0.00033292710,0.012465680,-
0.0023574670,-0.018121920,...
0.0060226430,0.025813150,-0.012422540,-
0.036906340,0.024020070,0.055379000,-
0.048731140,-0.098437790,...
0.13639810, 0.46139480]; % polovina koef.
filtra h0, filter je lin faze
h0=[h0,fliplr(h0)]; % filter je lin faza pa su koef.
simetricni
e0=h0(1:2:end);
e1=h0(2:2:end);
[E0,w]=freqz(upsample(e0,2),1,10000);
[E1,w]=freqz(upsample(e1,2),1,10000);
figure,plot(w/pi,4*abs(E0.*E1));
figure,plot(w/pi,angle(E0.*E1));
```



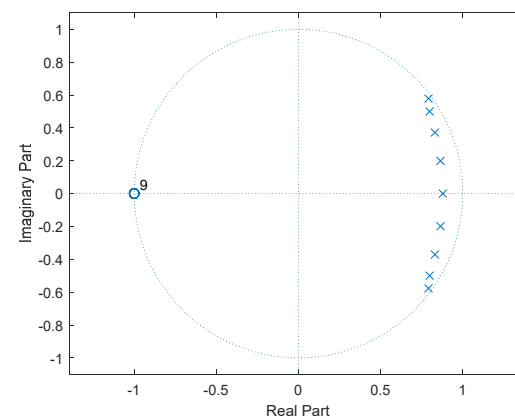
Dvokanalna IIR filtarska banka (*Half-band* IIR filtri)

- Realizacija preko paralelne veze svepropuniska (da bi se dobila efikasna struktura)
- Sekcije nižeg reda (drugog i prvog)
- Procesiranje na nižoj frekvenciji odabiranja

Realizacija preko sekcija nižeg reda



Podsetnik



$$G_{LP}(z) = [A_0(z) + A_1(z)]/2$$

$$G_{HP}(z) = [A_0(z) - A_1(z)]/2$$

$$\beta_l = (r_l)^2, \beta_l < \beta_{l+1}$$

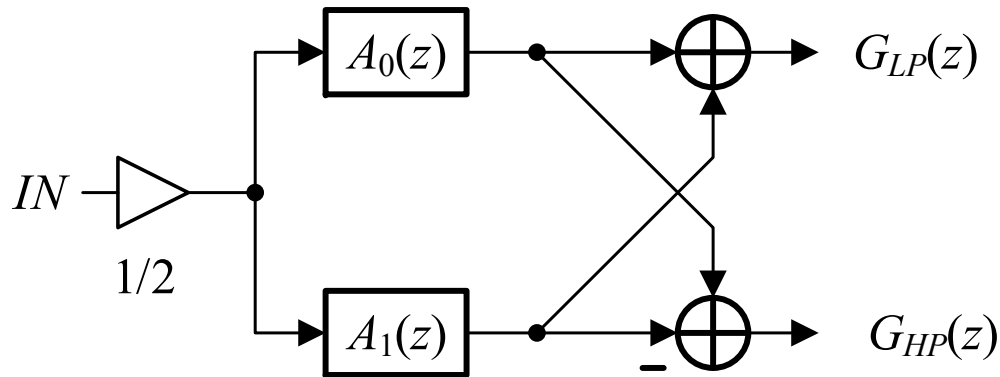
r_l – moduli
polova

$$A_0(z) = \prod_{l=2,4,\dots}^{(N+1)/2} \frac{\beta_l + \alpha_l(1 + \beta_l)z^{-1} + z^{-2}}{1 + \alpha_l(1 + \beta_l)z^{-1} + \beta_l z^{-2}}$$

$$A_1(z) = \frac{\alpha_1 + z^{-1}}{1 + \alpha_1 z^{-1}} \prod_{l=3,5,\dots}^{(N+1)/2} \frac{\beta_l + \alpha_l(1 + \beta_l)z^{-1} + z^{-2}}{1 + \alpha_l(1 + \beta_l)z^{-1} + \beta_l z^{-2}}$$

OPŠTI SLUČAJ – filtri **NE** moraju
biti *half-band*

Realizacija IIR filtarskog para preko paralelne veze *all-pass* filtara



$$G_{LP}(z) = [A_0(z) + A_1(z)]/2$$

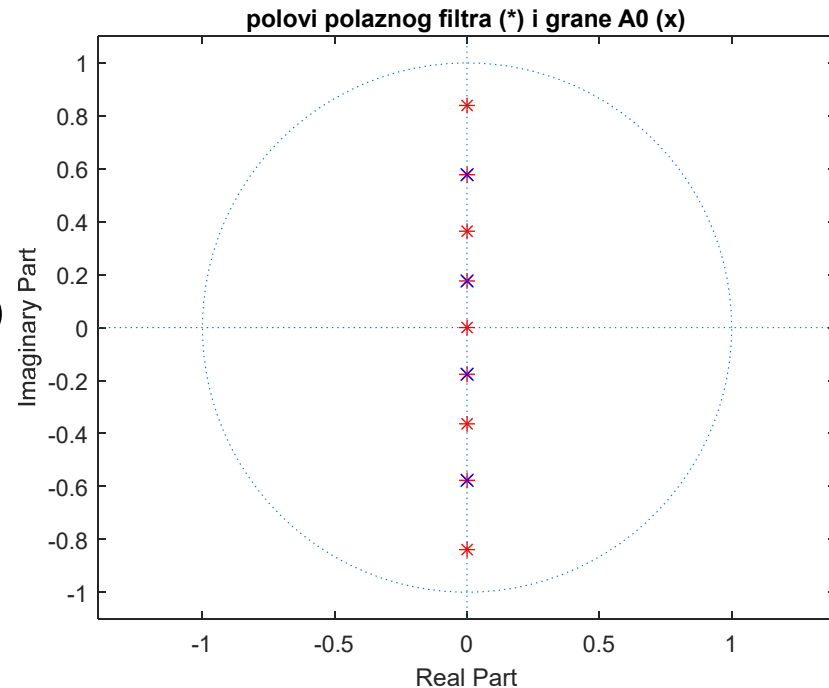
$$G_{HP}(z) = [A_0(z) - A_1(z)]/2$$

$$A_0(z) = \prod_{l=2,4,\dots}^{(N+1)/2} \frac{\beta_l + z^{-2}}{1 + \beta_l z^{-2}}$$

$$A_1(z) = z^{-1} \prod_{l=3,5,\dots}^{(N+1)/2} \frac{\beta_l + z^{-2}}{1 + \beta_l z^{-2}}$$

$$\beta_l = (r_l)^2, \quad \beta_l < \beta_{l+1}$$

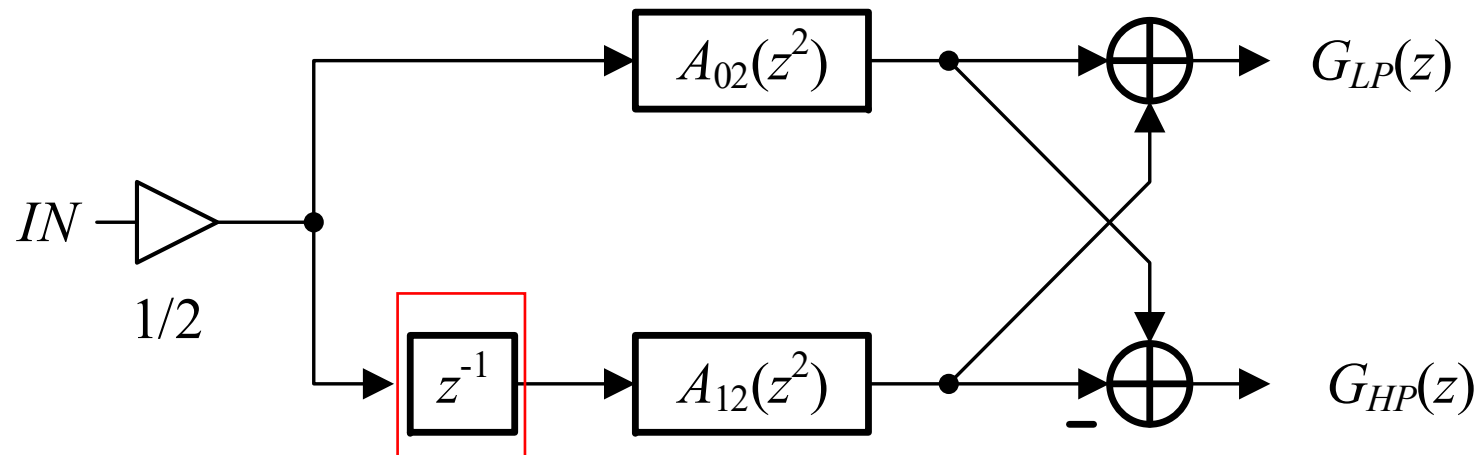
r_l – moduli
polova



Ako su filtri *half-band* moguće su dodatne
pogodne uštede u broju računskih operacija

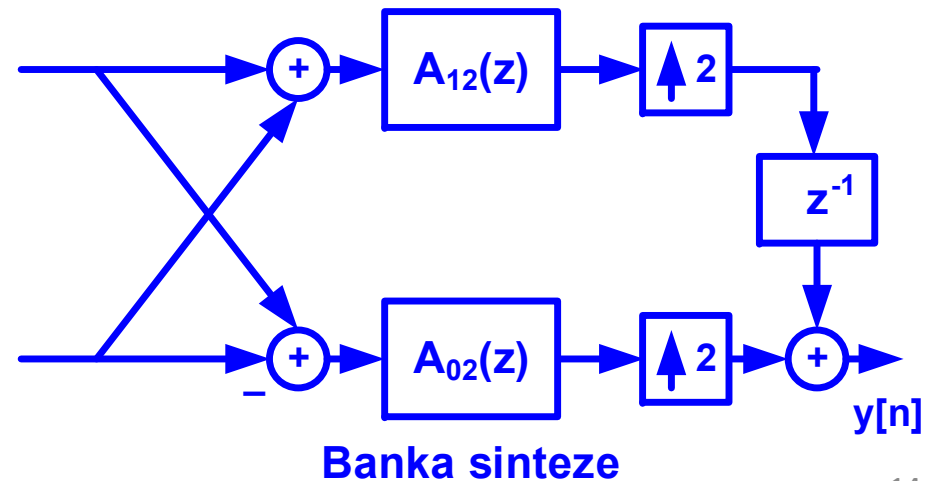
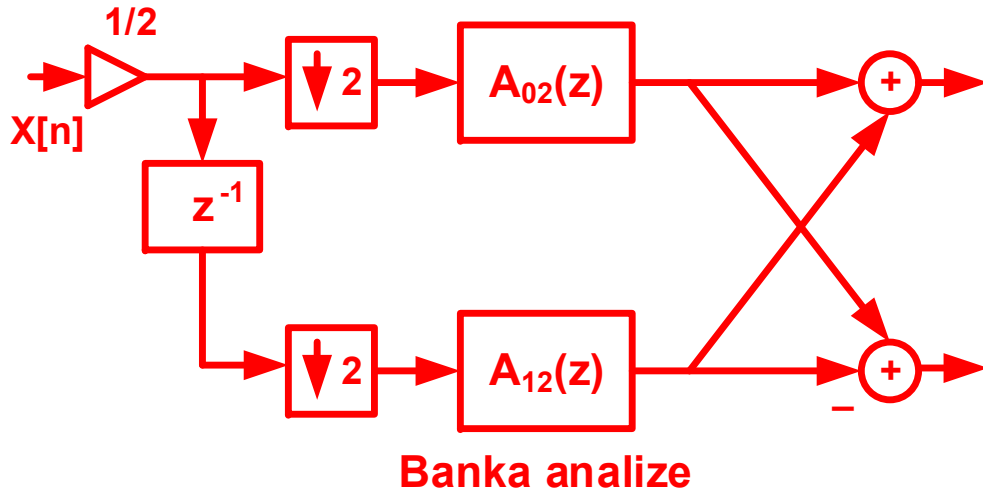
Half-band IIR filtri

Realizacija preko paralelne veze svepropusniska

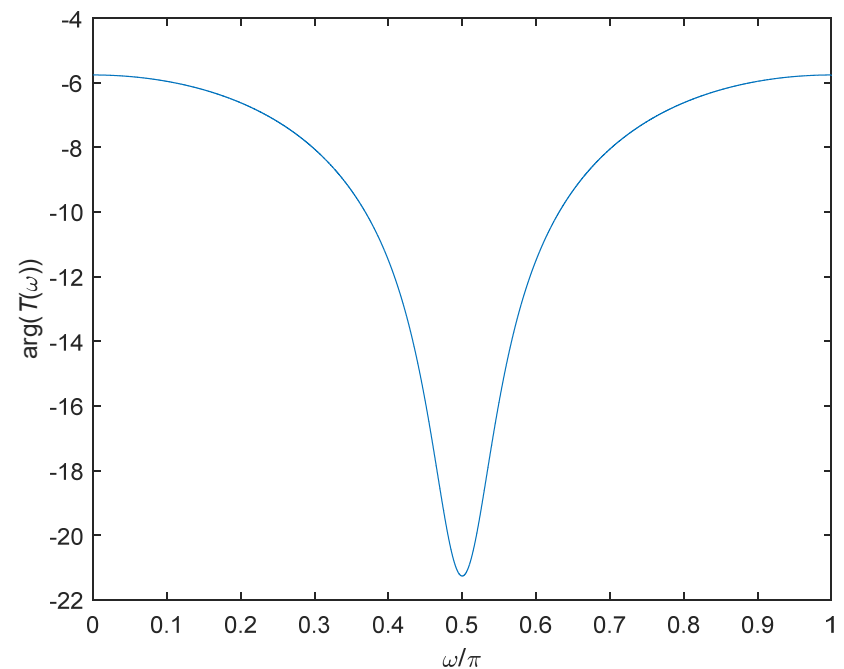
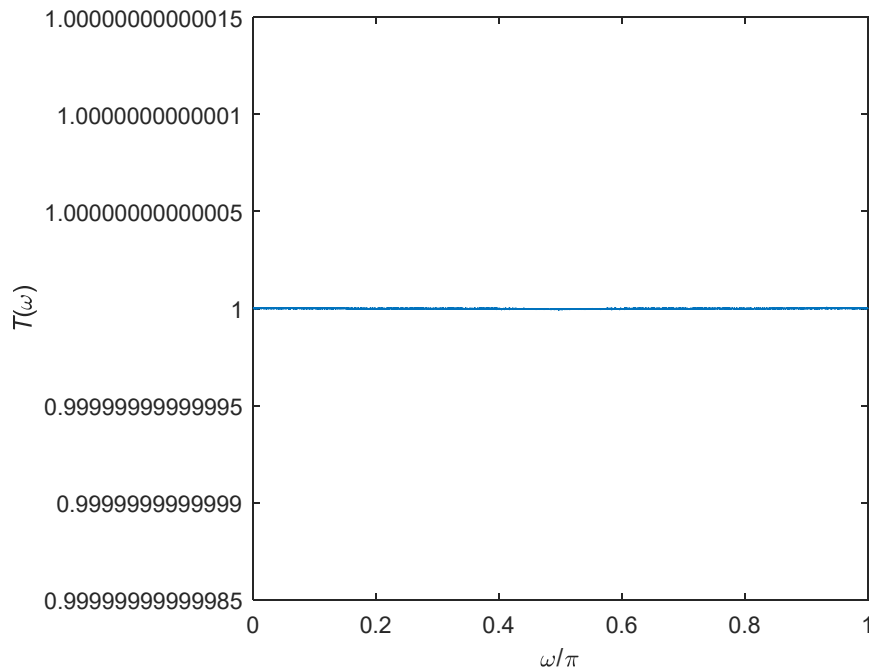


Jedinično kašnjenje – trivijalna sekcija prvog reda

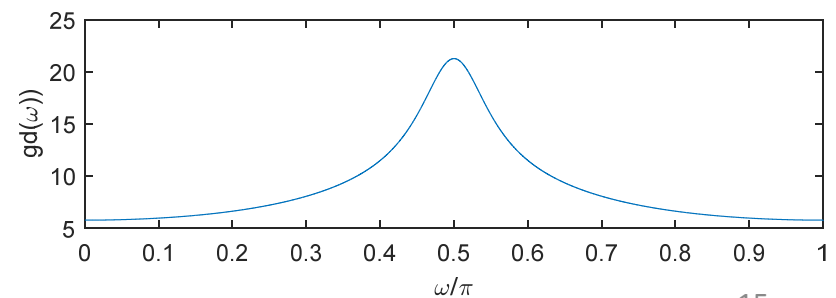
Dvokanalne banke filtera – efikasna realizacija – IIR



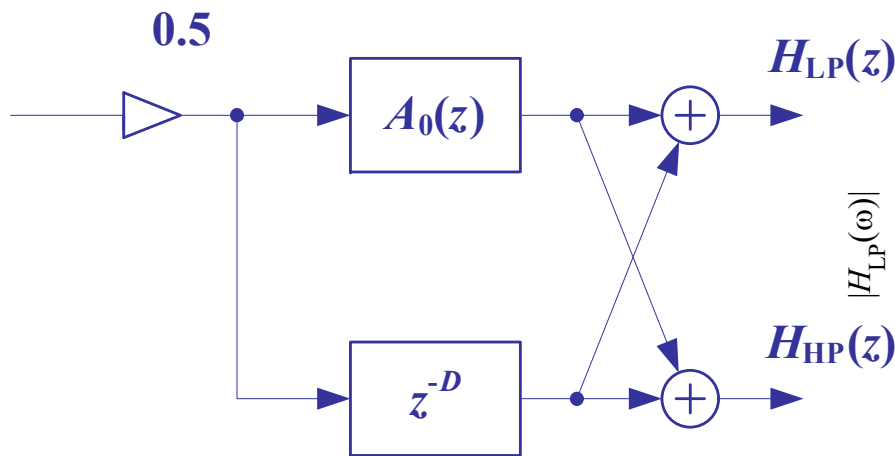
Dvokanalne banke filtra – efikasna realizacija – IIR



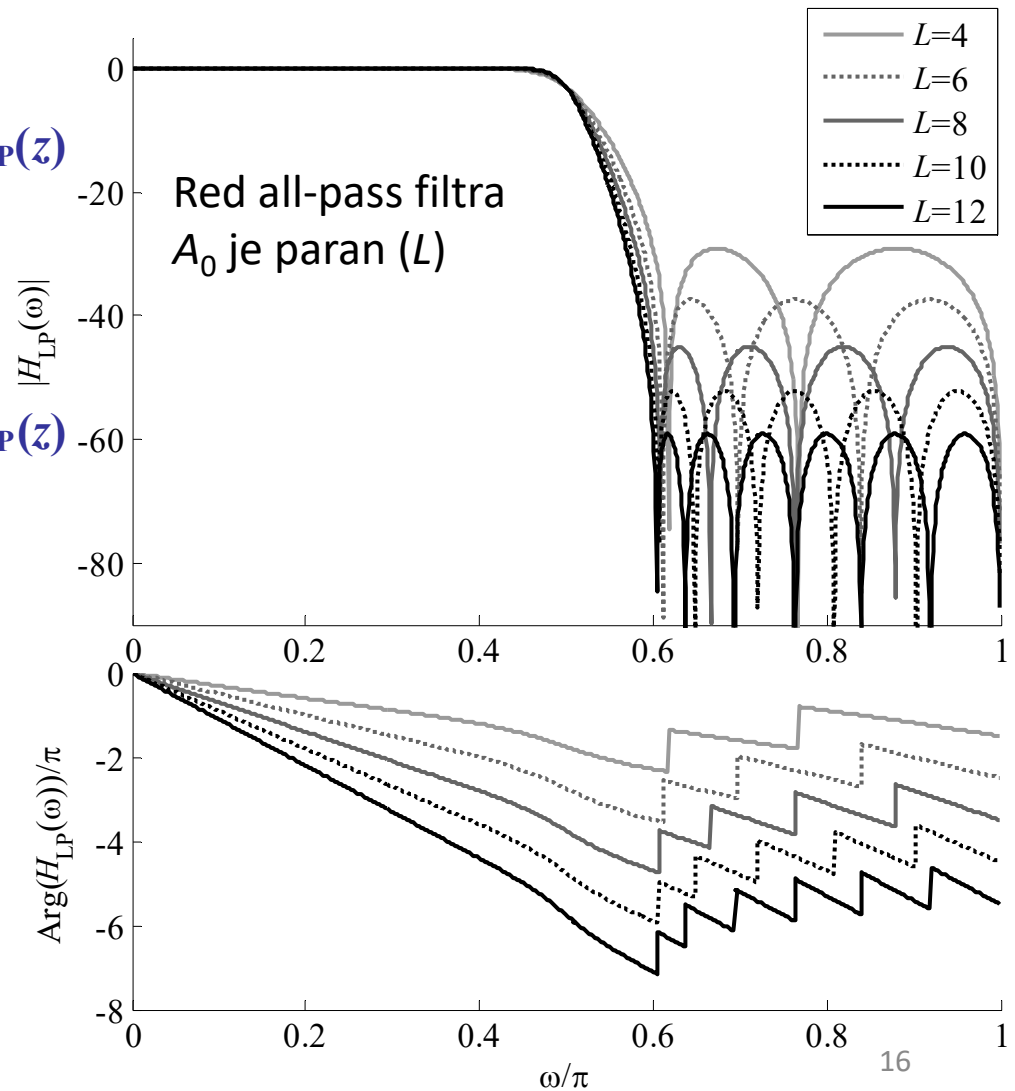
NIJE banka sa perfektnom
rekonstrukcijom (ni blizu, jer nema
linearnu fazu)



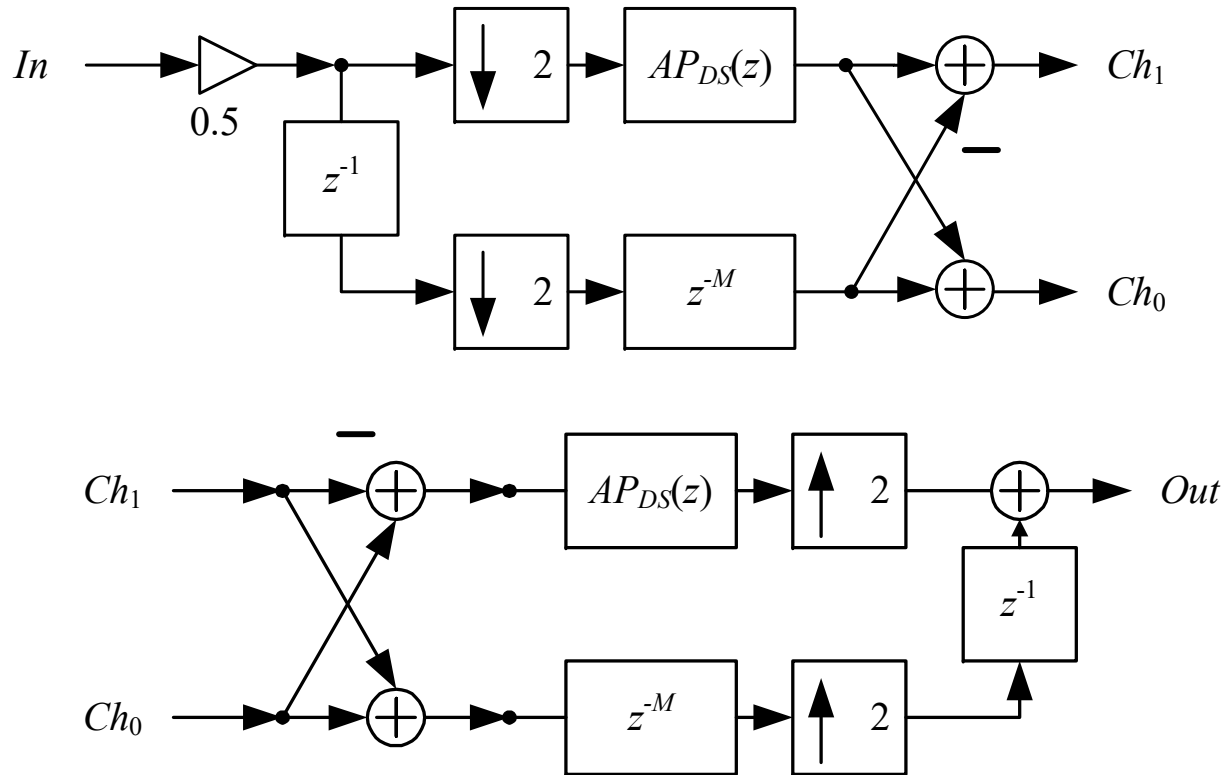
IIR filtri približno linearne faze



Paralelna veza *all-pass* filtra i kašnjenja (koje je trivijalna *all-pass* sekcija), pa je ovo specijalan slučaj filtra realizovanog kao paralelna veza dva svepropusnika

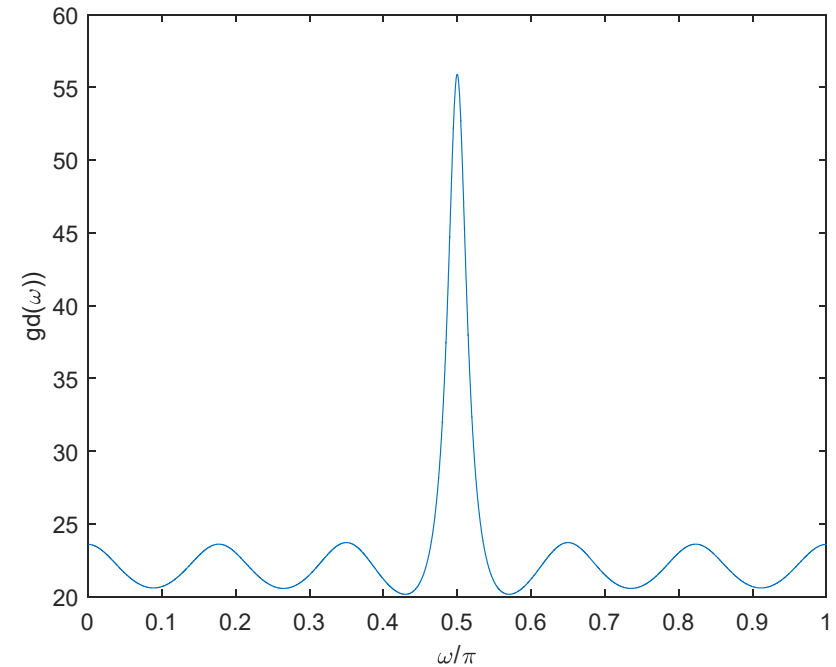
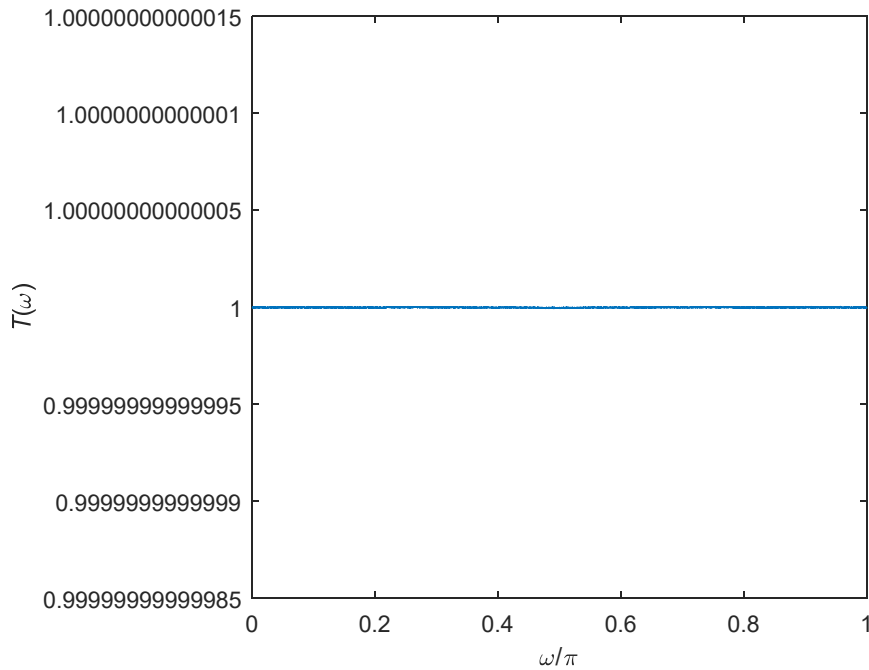


IIR filtri približno linearne faze filtarska banka

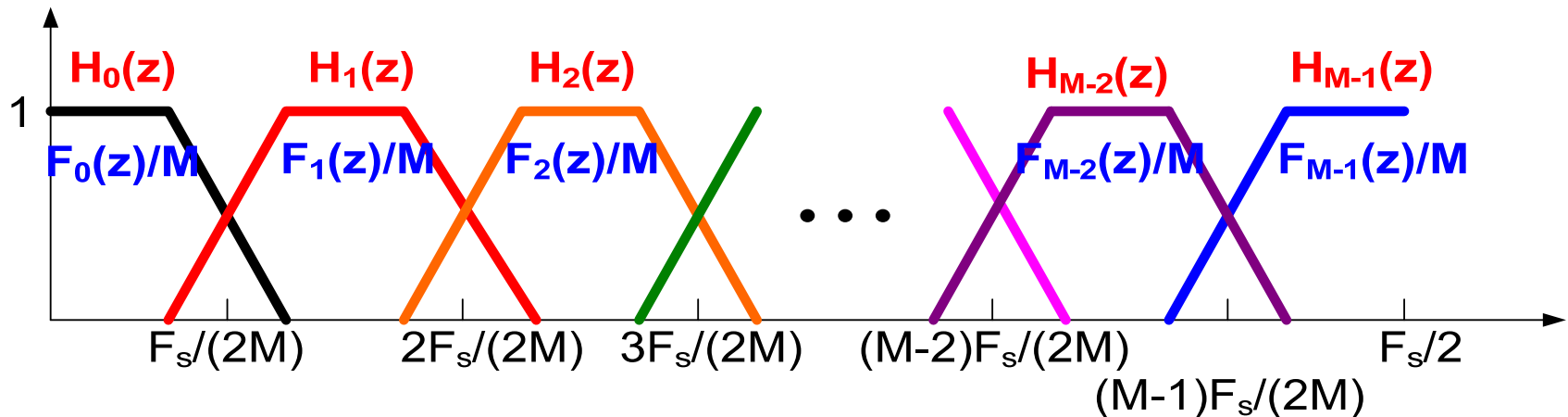


$$A_0(z) = AP_{DS}(z^2)$$

IIR filtri probližno linearne faze

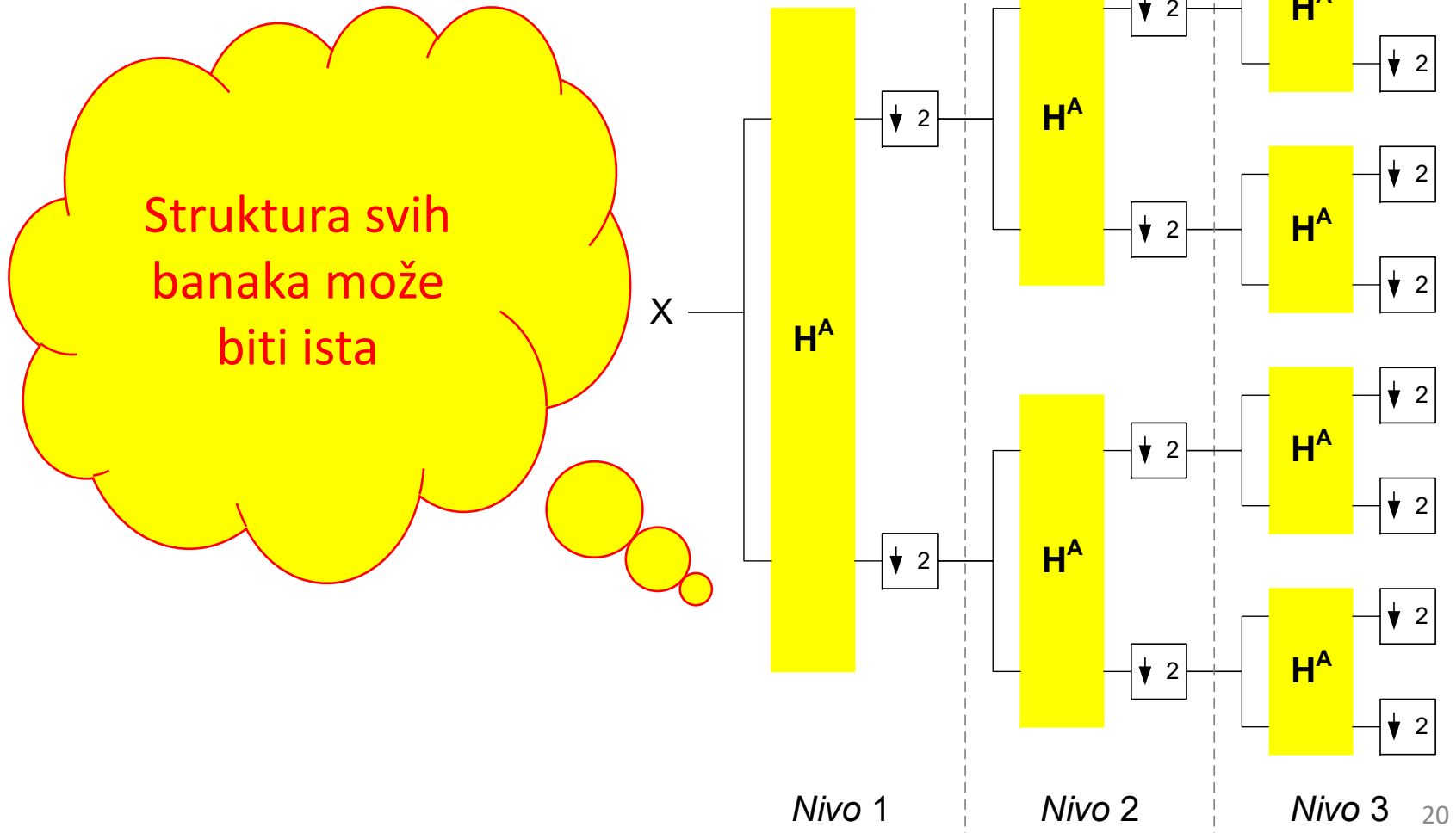


Uniformna banka filtara

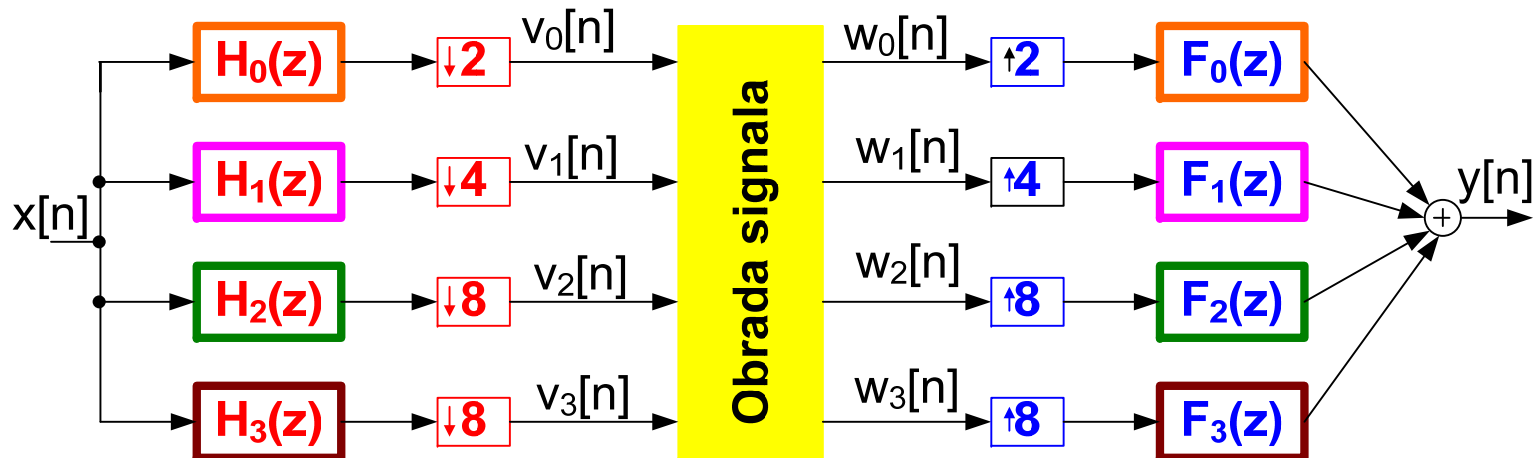
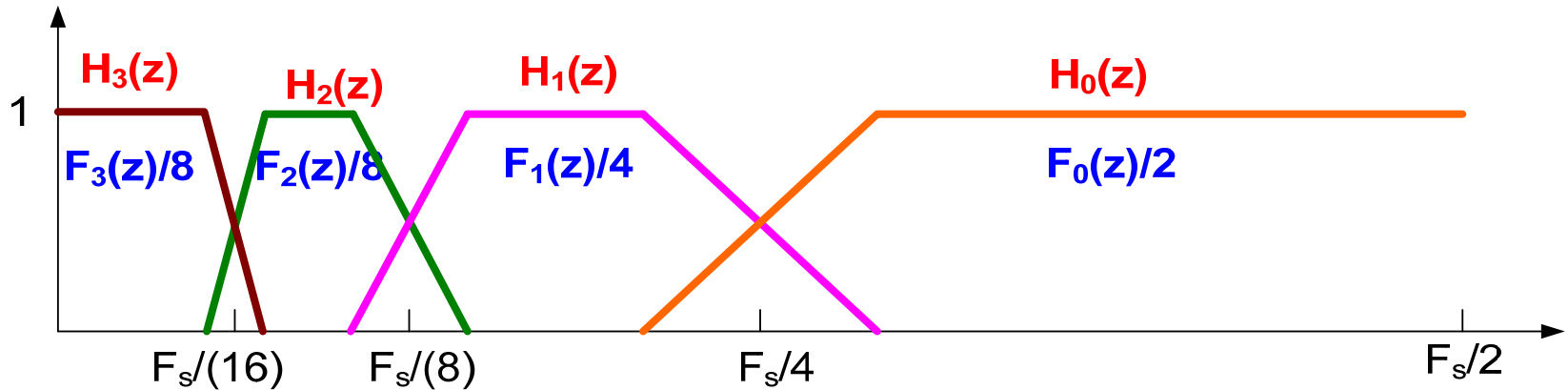


- filtri $H_0(z)$ i $F_0(z)$ - propusnici niskih frekvencija
- filtri $H_{M-1}(z)$ i $F_{M-1}(z)$ propusnici visokih frekvencija
- Ostali filtri su propusnici opsega

Realizacija preko dvokanlanih banaka

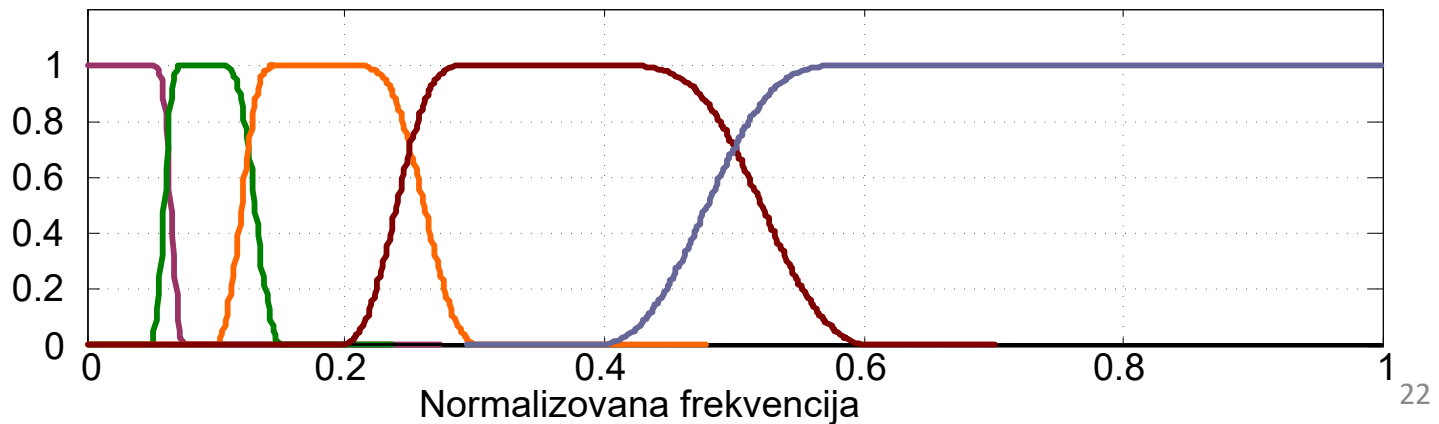
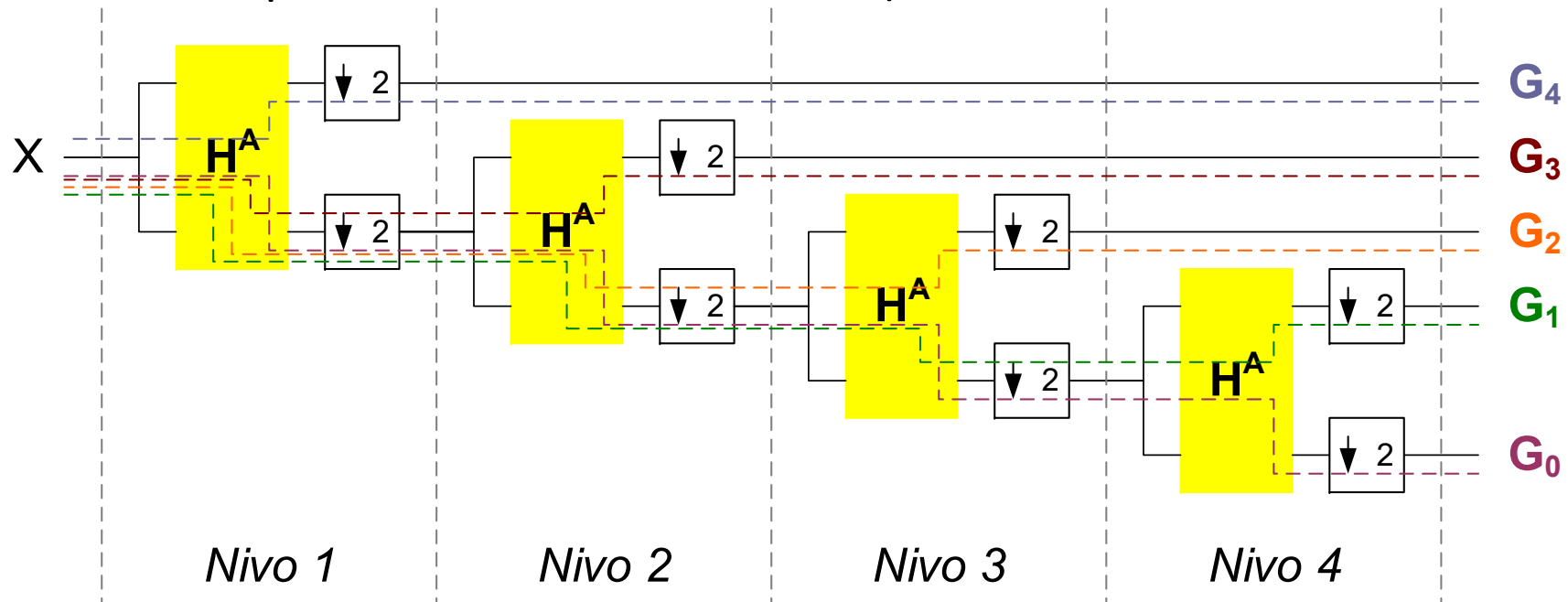


Oktavna banka filtera (neuniformna)



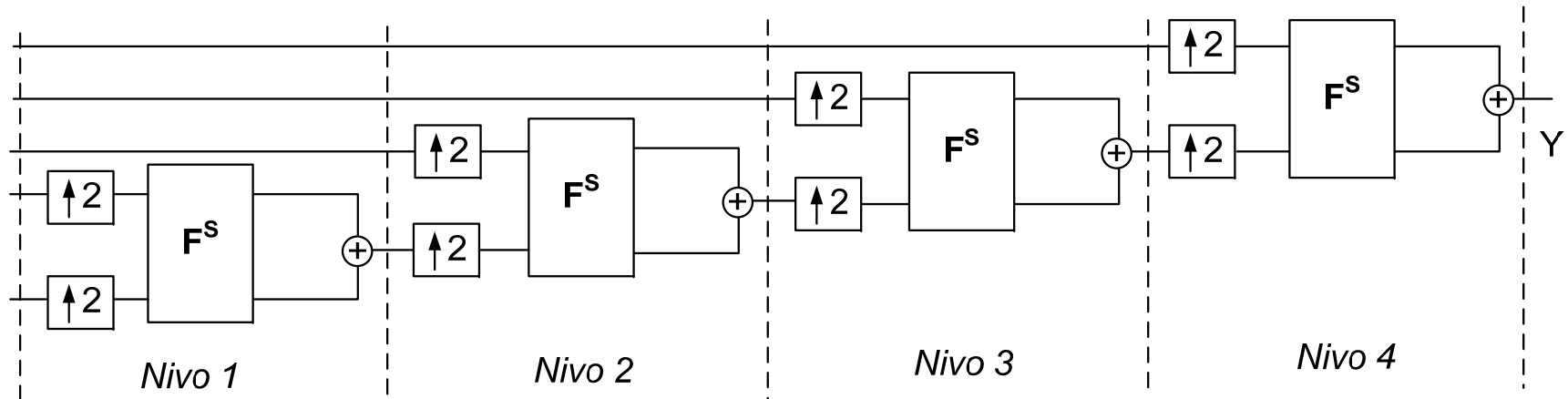
Oktavna banka analize

(realizovana pomoću dvokanalnih banki)



Oktavna banka sinteze

(realizovana pomoću dvokanalnih banki)



Direktna realizacija višekanalnih uniformnih banki filtara

- Direktna sinteza višekanalnih filtarskih banaka (posebno ako se zahteva približno savršena rekonstrukcija) može se postići optimizacionim metodama
- Ovakav način projektovanja je složen i ograničen na relativno mali broj kanala
- U praksi su od interesa filtarske banke koje spadaju u klasu filtarskih banaka sa kompleksnom modulacijom

Filterske banke sa kompleksnom modulacijom

U opštem slučaju, filter propusnik opsega H_2 može se realizovati „transliranjem u spektru“ (kompleksnom modulacijom) filtra propusnika niskih frekvencija H_1

$$h_2[n] = h_1[n] e^{jn\omega_0}$$

$$H_2(e^{j\omega}) = H_1(e^{j(\omega-\omega_0)})$$

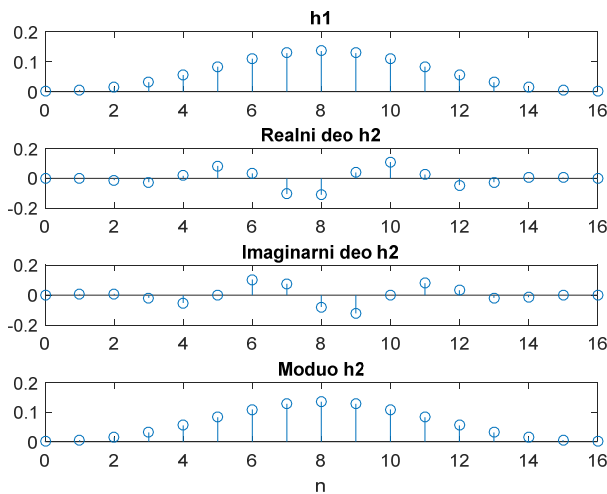
Iako je filter H_1 filter sa realnim koeficijentima, filter H_2 je, očigledno, filter sa kompleksnim koeficijentima

Ukoliko H_1 ima linearnu faznu karakteristiku, i H_2 će imati linearnu faznu karakteristiku

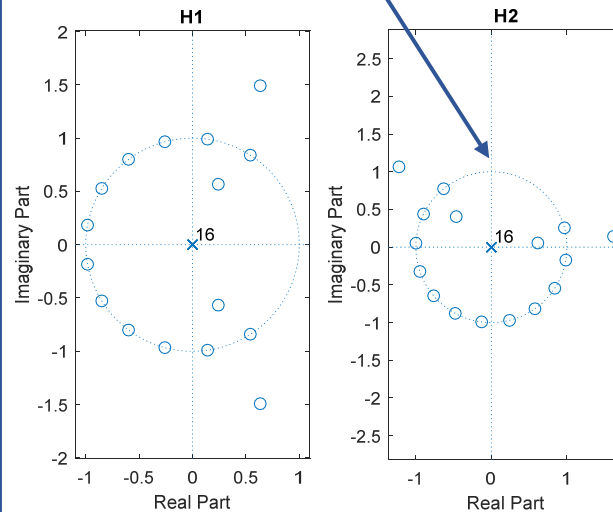
Filterske banke sa kompleksnom modulacijom

$$h_2[n] = h_1[n]e^{jn\omega_0}$$

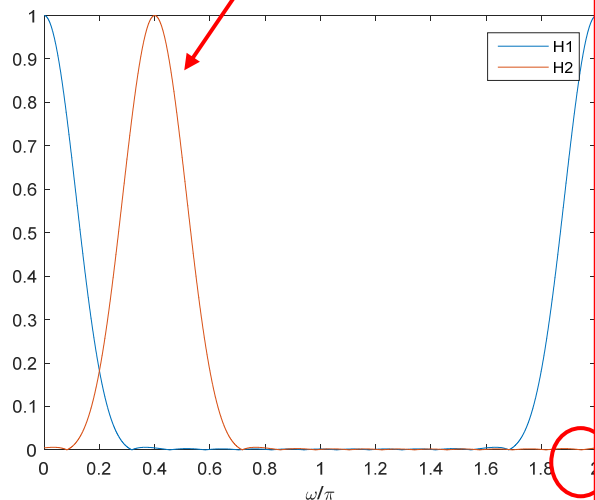
$$H_2(e^{j\omega}) = H_1(e^{j(\omega-\omega_0)})$$



Položaj nula i polova se „zarotirao“ za ugao ω_0

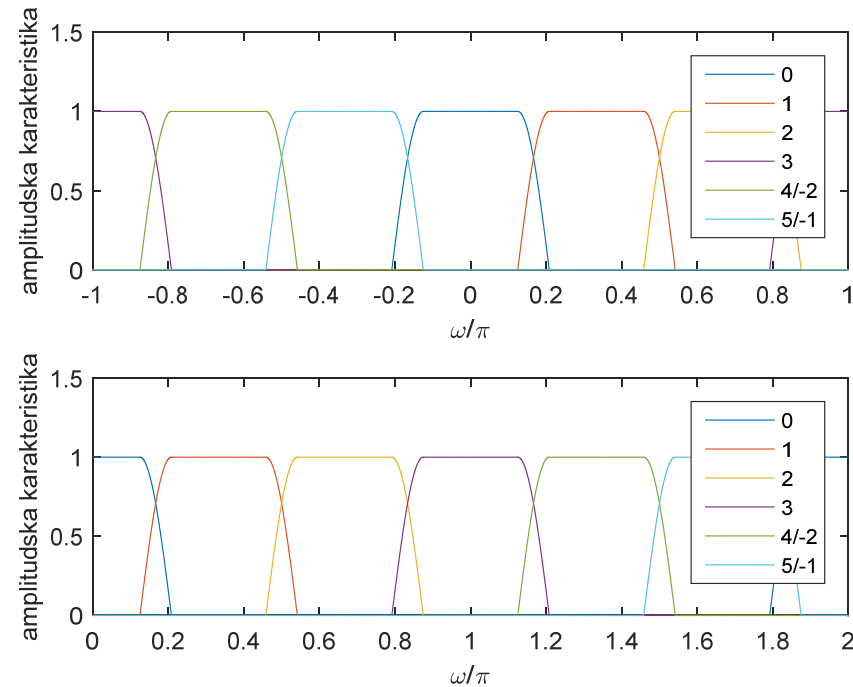


Hrekvencijski odziv se „translirao“ za ω_0
 H_2 je sistem sa kompleksnim koeficijentima NEMA simetrije oko 0



Direktna realizacija višekanalnih uniformnih banki filtara

Ovaj princip može da se iskoristi tako što se ista karakteristika filtra propusnika niskih frekvencija translira M puta sa različitim „pomacima“ u spektru $\omega_k = 2\pi k/M$, $k=0,1,2,\dots,M-1$ gde je M ukupan broj kanala na intervalu od 0 do 2π



Direktna realizacija višekanalnih uniformnih banki filtera - DFT filterarske banke

• DFT banke

- M -kanalna DFT banka formira se od prototip filtera propusnika niskih frekvencija $H_p(z)$ i skupa od M modulatora. Ako su koeficijenti impulsnog odziva NF prototip filtera N -tog reda: $h_0(0), h_0(1), \dots, h_0(N)$, tada će koeficijenti impulsnog odziva k -tog filtera u M -kanalnoj banci biti:

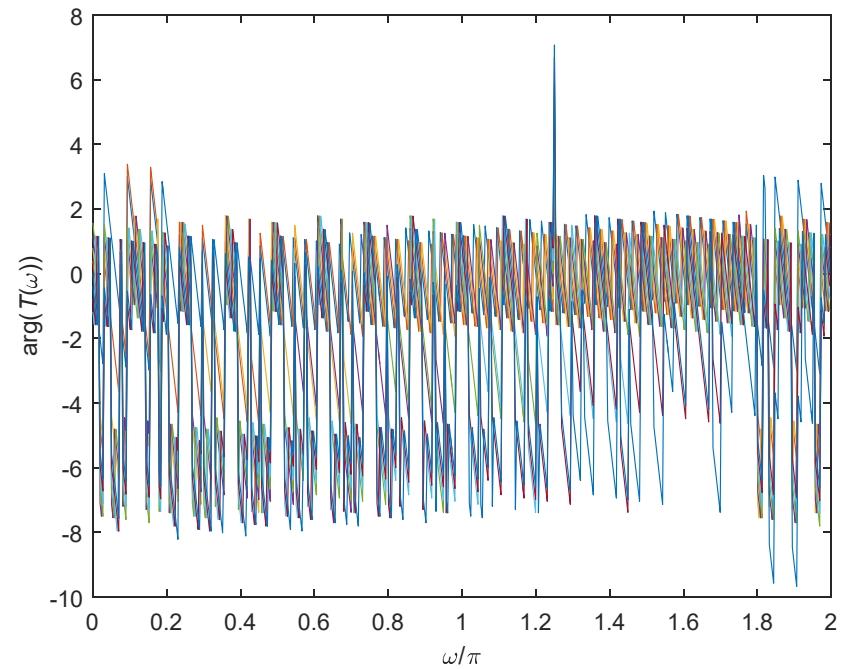
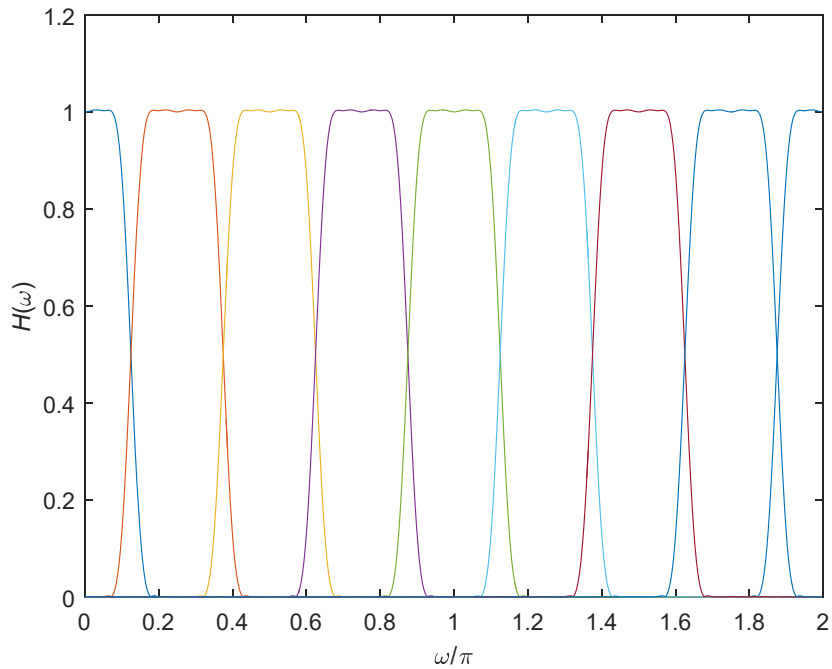
$$h_k[n] = f_k[n] = h_p[n]e^{j2\pi kn/M}, \text{ za } n = 0, 1, \dots, N$$

- Odgovarajući frekvencijski odziv k -tog filtera u banci je:

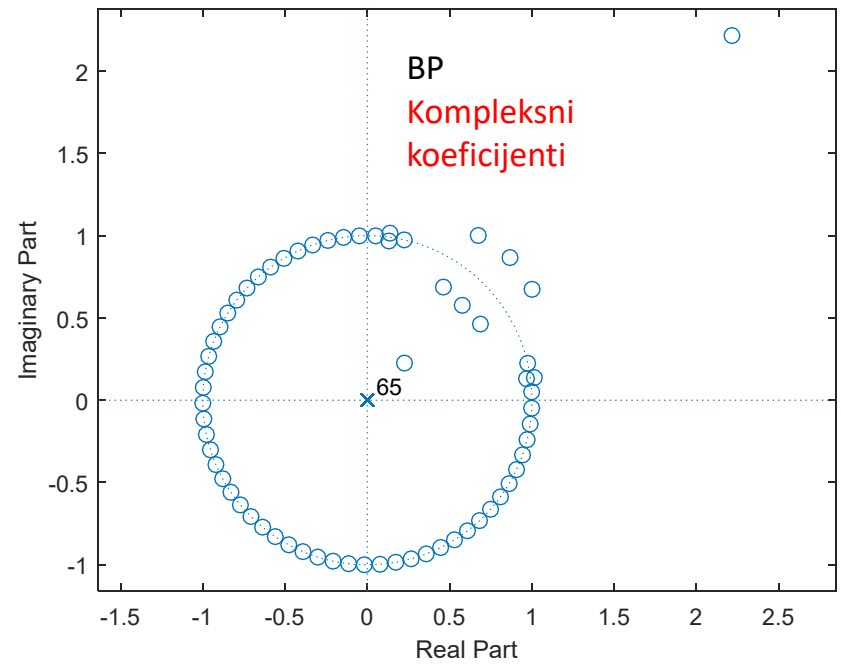
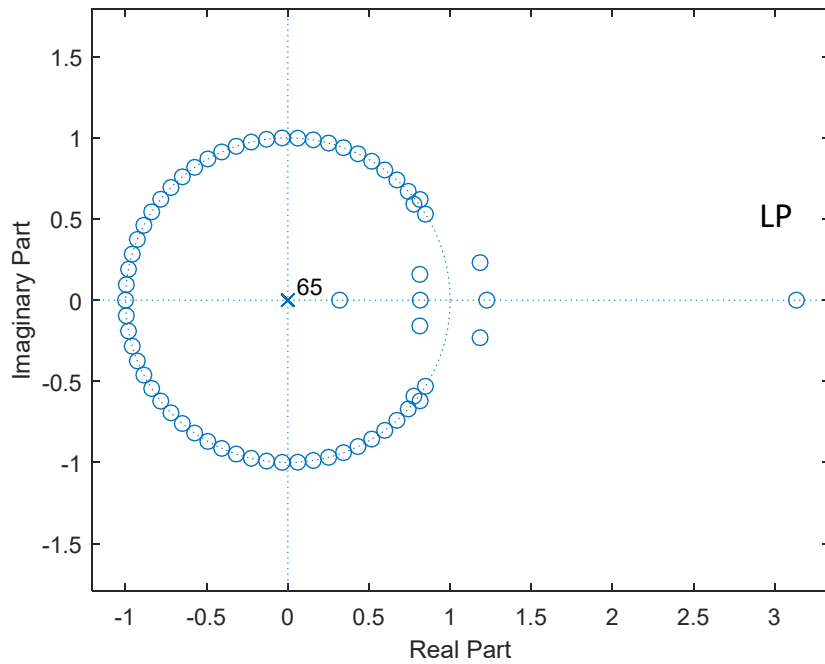
$$H_k(e^{j\omega}) = F_k(e^{j\omega}) = H_0(e^{j(\omega - 2\pi k/M)}), \text{ za } k = 0, 1, \dots, M - 1$$

- frekvencijski odzivi $H_k(e^{j\omega}), F_k(e^{j\omega}), k=1, 2, \dots, M-1$, su uniformno translirane verzija frekvencijskog odziva prototip filtera $H_0(e^{j\omega})$.

DFT banke

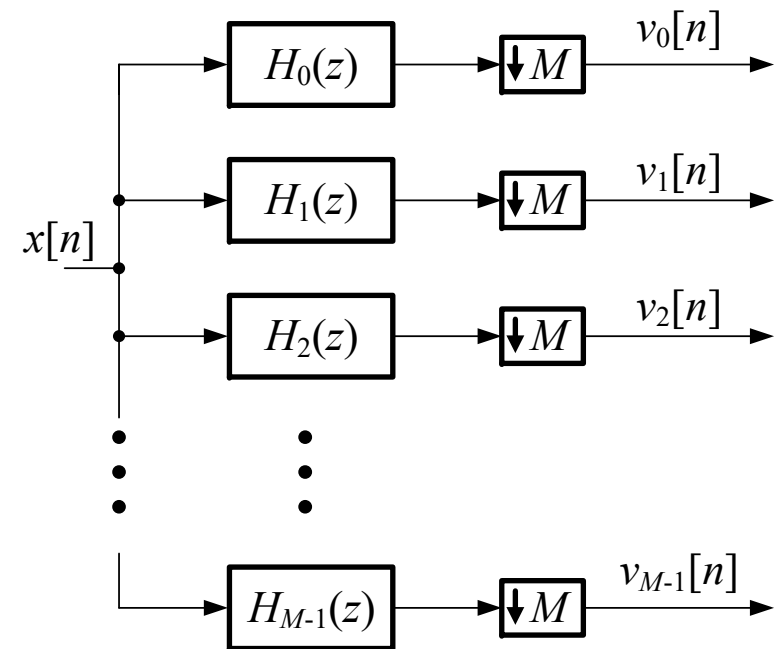


DFT banke

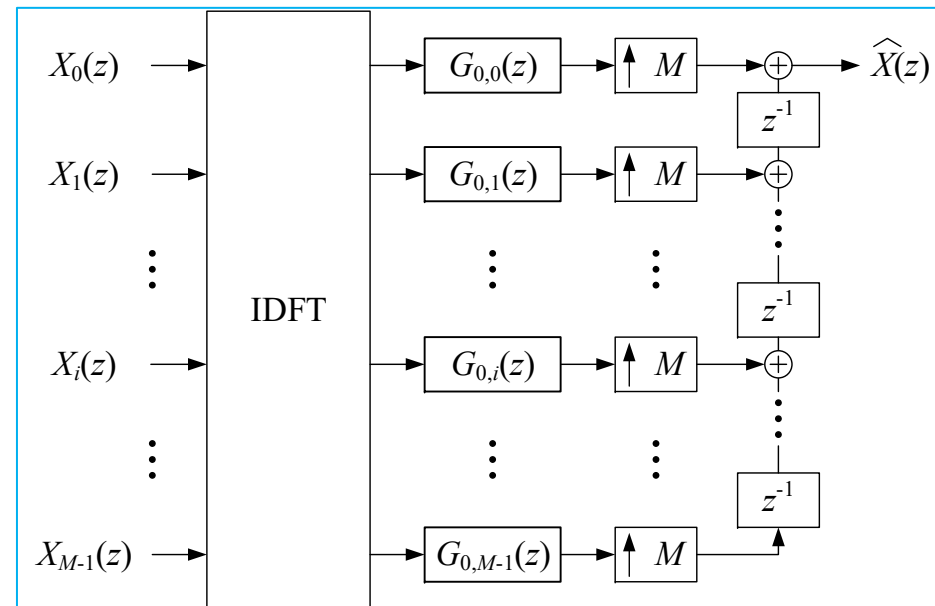
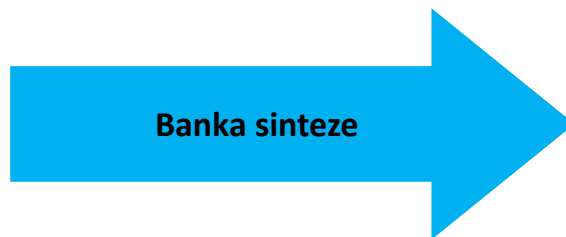
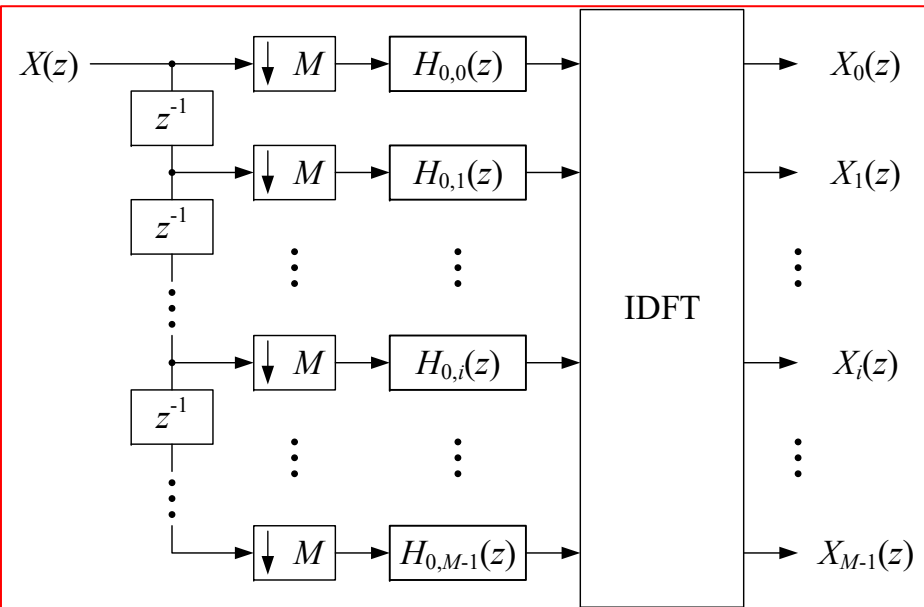


DFT banke

- Ako se opisana filtarska banka realizuje tako što se svaki filter implementira kao polifazna struktura, zbog veze između filtera, može se dobiti veoma efikasna struktura
- Efikasnost proističe iz toga što se dft/idft može iskoristiti kao blok za transliranje u spektru



DFT banke



DFT banke

- Iako je struktura veoma efikasna, postoji problem sa preklapanjem spektra između susednih kanala
- Sam prototip filter se projektuje tako da je kvadrat njegove amplitudske karakteristike podignuti kosinus, odnosno filter je “koren iz podignutog kosinusa” (*square root raised cosine*) **SRRC**
- Takav filter je komplementaran po snazi sa sopstvenim pomerenim replikama (transliranim) što bi obezbedilo da ukupna karakteristika filtera od ulaza u banku analize do izlaza iz banke sinteze odgovara kašnjenju
- Filter je idealan, pa se, odsecanjem, aproksimira FIR filtrom, sa dodatnim eventualnim množenjem sa prozorskom funkcijom

SRRC filter

Frekvencijski odziv

$$H_{SRRC}(e^{j\omega}) = \begin{cases} 1, & \frac{|\omega|}{\omega_c} \leq 1-r \\ \cos\left(\frac{\pi}{4r}\left(\frac{\omega}{\omega_c} - (1-r)\right)\right), & 1-r \leq \frac{|\omega|}{\omega_c} \leq 1+r \\ 0, & \frac{|\omega|}{\omega_c} \geq 1+r \end{cases}$$

Nekauzalan filter beskonačne dužine
impulsnog odziva

Aproksimira se FIR filtrom (kauzalnim) koji se
dobija metodom projektovanja pomoću
prozora, polazeći od datog impulsnog odziva

Impulsni odziv

$$h_{SRRC}[n] = \frac{4rn \cos\left(\frac{\pi n(1+r)}{M}\right) + M \sin\left(\frac{\pi n(1-r)}{M}\right)}{\left(1 - \left(\frac{4rn}{M}\right)^2\right) \pi n M}$$

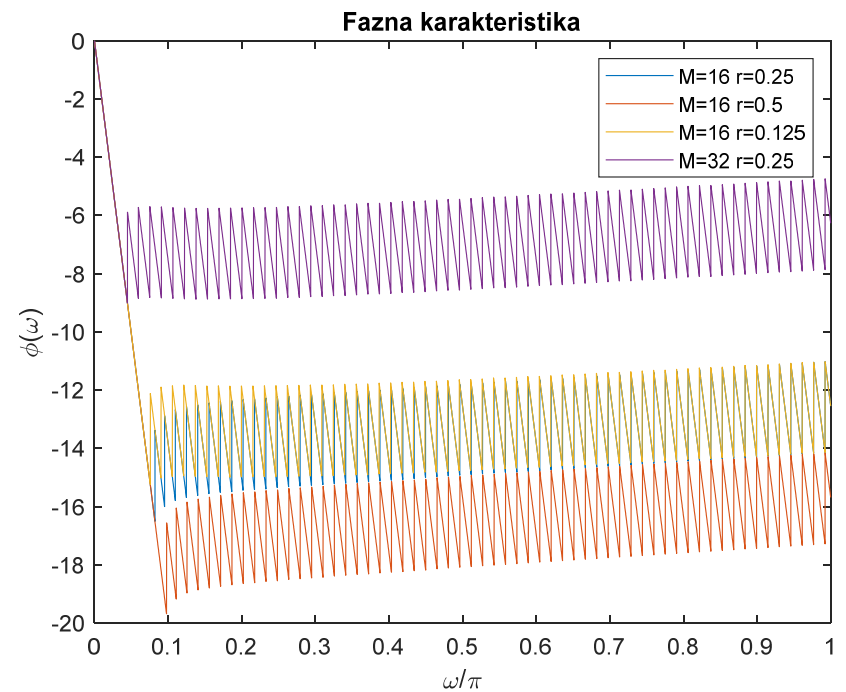
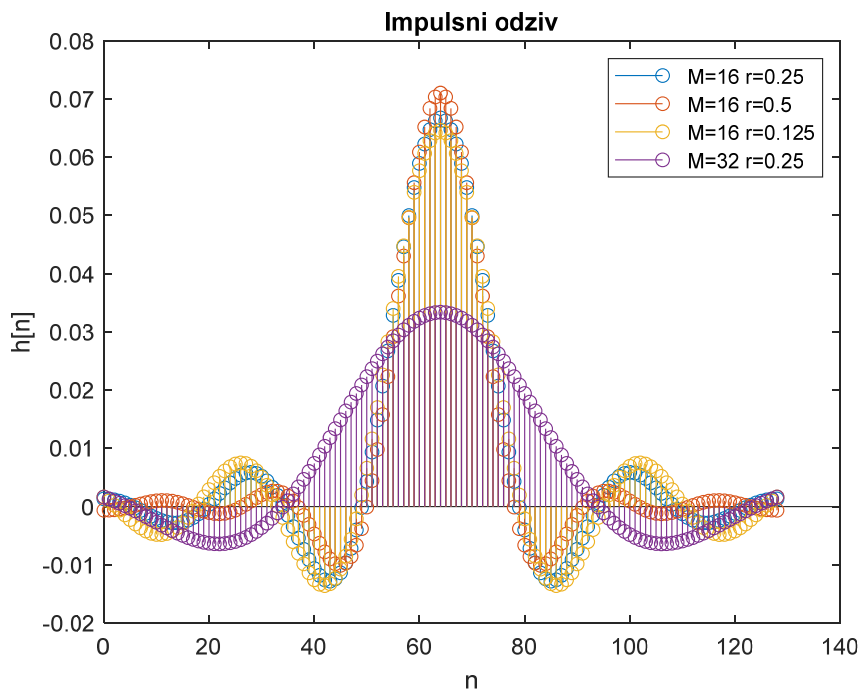
Za elemente impulsnog odziva za koje je
 $n=M/(4r)$, ukoliko postoje (ako je $M/(4r)$ ceo
broj)

$$h_{SRRC}\left[\pm \frac{M}{4r}\right] = -\frac{r}{M} \left(\frac{2}{\pi} \cos\left(\frac{\pi}{4r}(1+r)\right) - \cos\left(\frac{\pi}{4r}(1-r)\right) \right)$$

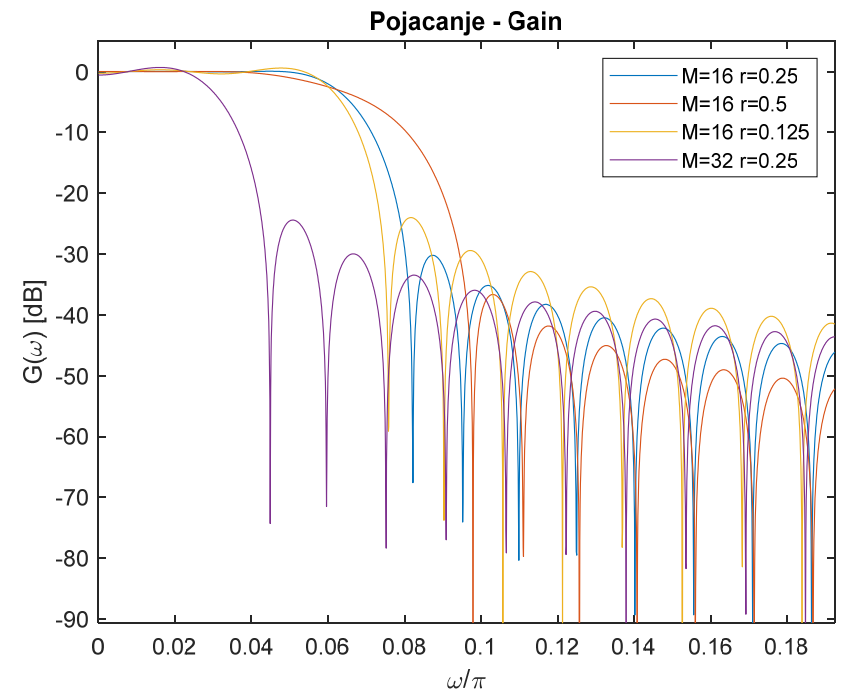
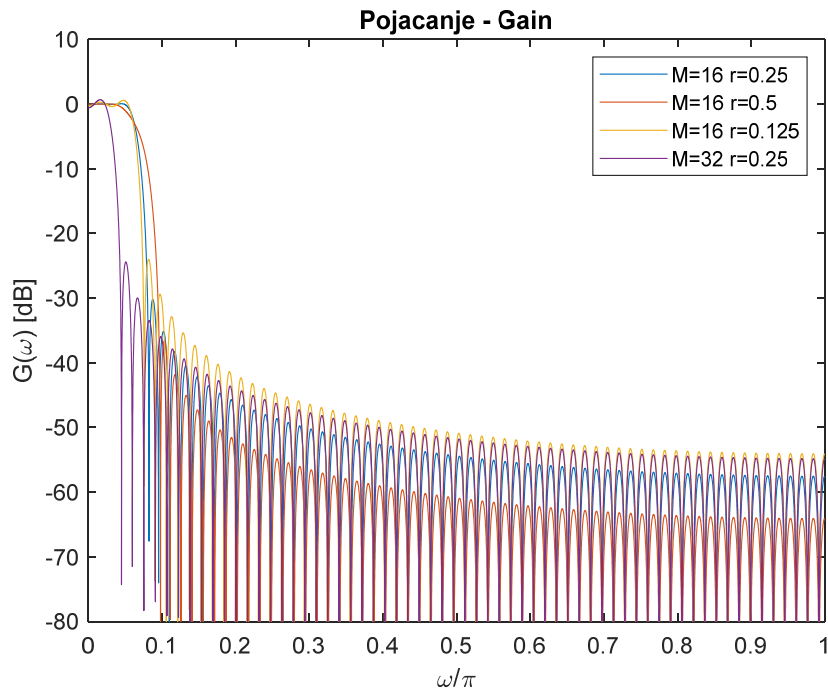
za $n=0$

$$h_{SRRC}[0] = \frac{1}{M} + \frac{r}{M} \left(\frac{4}{\pi} - 1 \right)$$

SRRC filtar



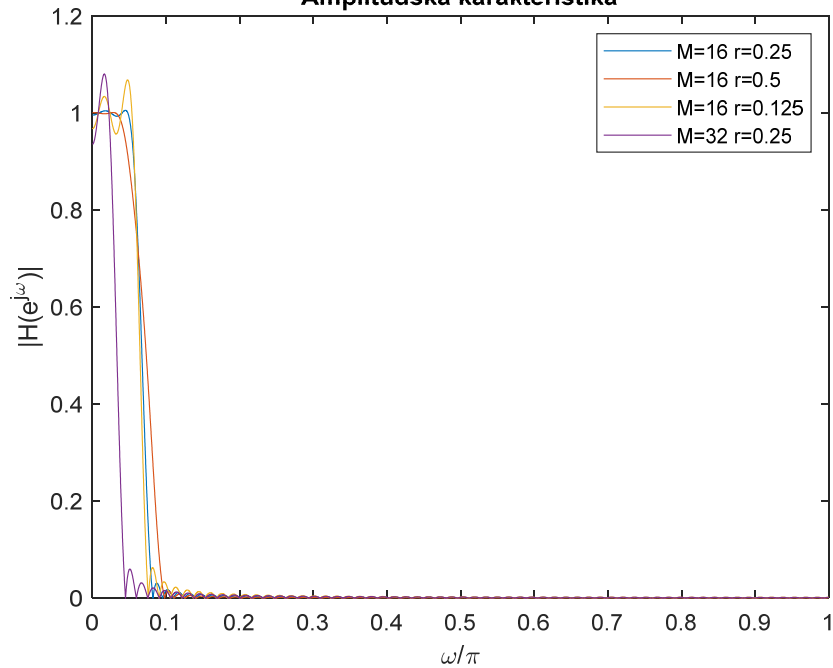
SRRC filtar



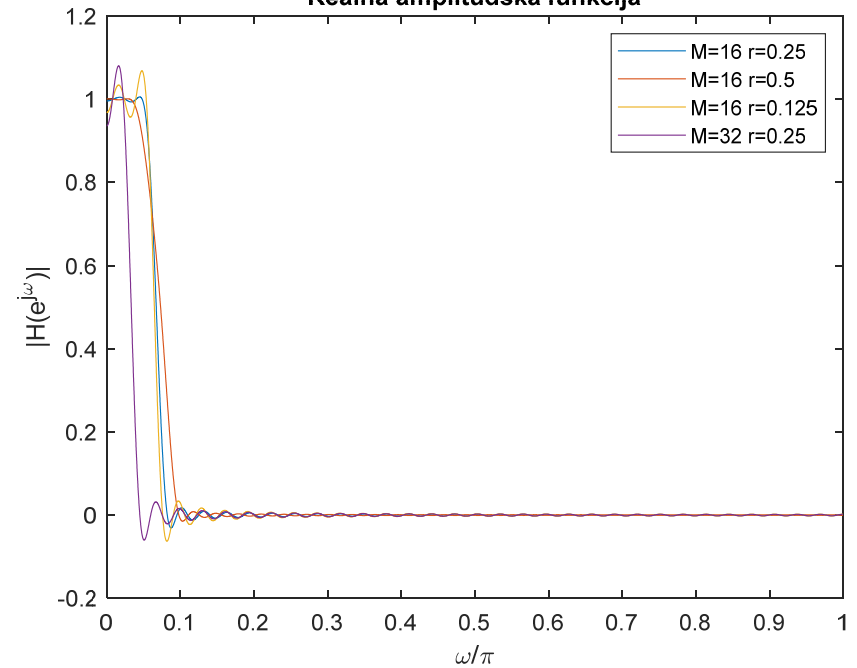
Zoom – propusni opseg i
prelazna zona

SRRC filtar

Amplitudska karakteristika



Realna amplitudska funkcija



DFT banke

U osnovnoj realizaciji, problem je preklapanje spektra koje ne može potpuno da se eliminiše, postoje razne modifikacije koje to delimično rešavaju, više o tome u *

*T. Karp and N. J. Fliege, "Modified DFT filter banks with perfect reconstruction," in *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 46, no. 11, pp. 1404-1414, Nov. 1999.

Direktna realizacija višekanalnih uniformnih banki filtera – kosinusne banke

Kosinusne banke su sa realnim koeficijentima, M kanala u opsegu od 0 do π

- **Kosinusne banke**

- Kosinusne banke filtera formiraju se takođe iz prototip NF filtera.
- Koeficijenti impulsnog odziva filtera u kosinusnoj banci formiraju se od koeficijenata impulsnog odziva NF prototip filtera i skupa kosinusnih modulatora prema izrazu:

$$h_k[n] = 2p_0[n] \cos\left(\left(n - \frac{N}{2}\right) \frac{(2k+1)\pi}{2M} + (-1)^k \frac{\pi}{4}\right), \text{ za } k = 0, 1, \dots, M-1 \quad \text{Banka analize}$$

$$f_k[n] = 2p_0[n] \cos\left(\left(n - \frac{N}{2}\right) \frac{(2k+1)\pi}{2M} - (-1)^k \frac{\pi}{4}\right), \text{ za } k = 0, 1, \dots, M-1 \quad \text{Banka sinteze}$$

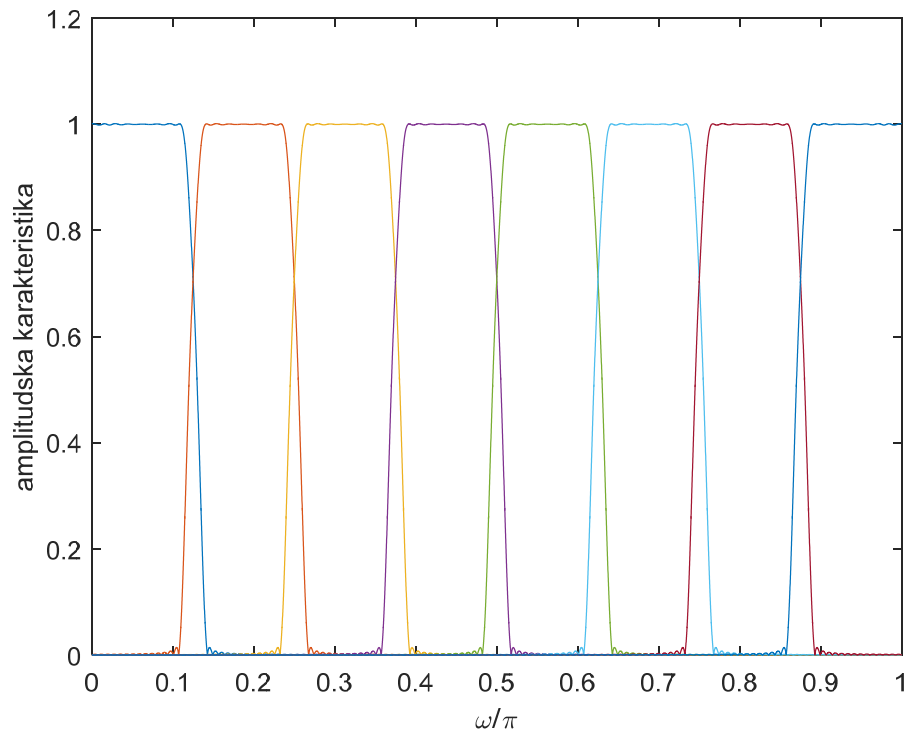
Direktna realizacija višekanalnih uniformnih banki filtera – kosinusne banke

- Kanalni filteri nisu filteri linearne faze
- Filteri u banci sinteze nisu isti kao filteri u banci analize
- Ukupna karakteristika je linearna

$$h_k[n] = 2p_0[n] \cos\left(\left(n - \frac{N}{2}\right) \frac{(2k+1)\pi}{2M} + (-1)^k \frac{\pi}{4}\right), \text{ za } k = 0, 1, \dots, M-1 \quad \text{Banka analize}$$

$$f_k[n] = 2p_0[n] \cos\left(\left(n - \frac{N}{2}\right) \frac{(2k+1)\pi}{2M} - (-1)^k \frac{\pi}{4}\right), \text{ za } k = 0, 1, \dots, M-1 \quad \text{Banka sinteze}$$

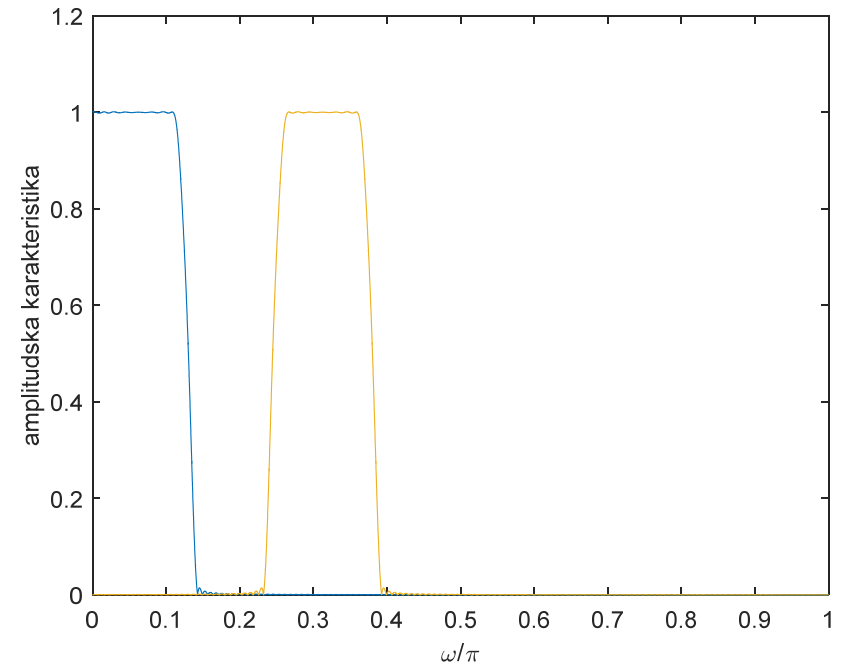
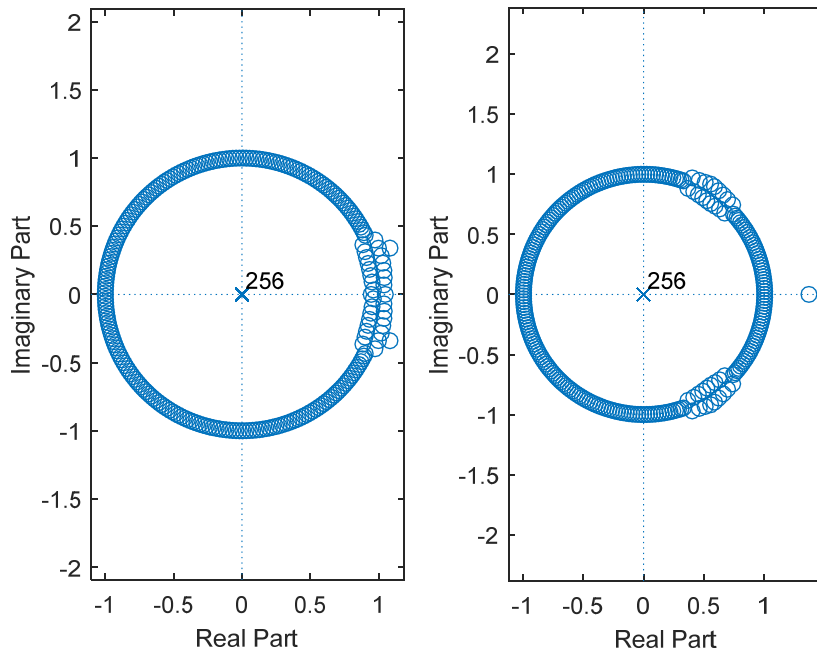
Kosinusne banke



Filtri su sa realnim koeficijentima, prototip filter mora biti dva puta manje granične frekvencije, odnosno $\pi/(2M)$

Filtri treba da budu komplementarni po snazi

Kosinusne banke

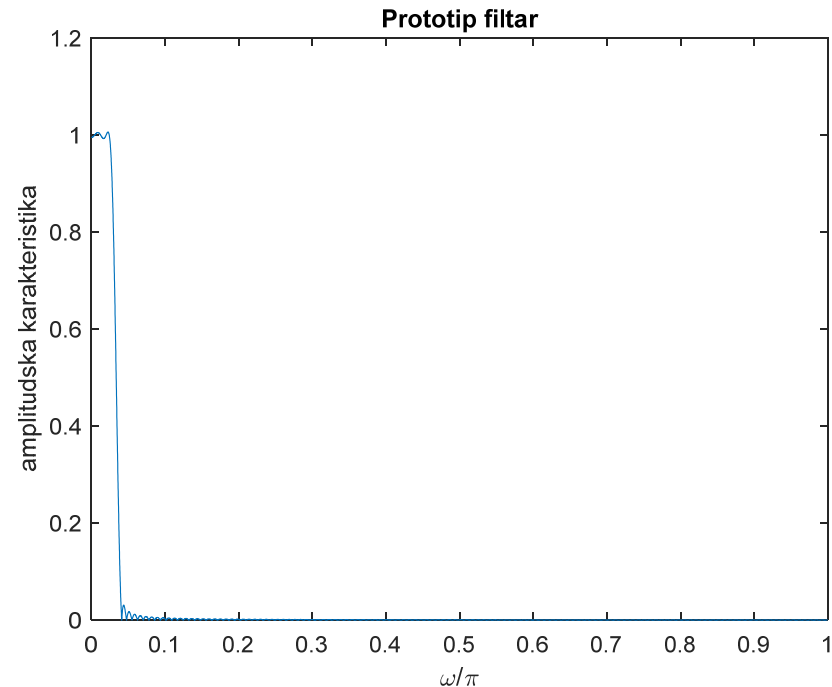


Kosinusna banka - primer

```

close all
clear
clc
M=16;
% filtarska banka
N=256;
Nx=(N+1)*M;
h0=srrcf(N,2*M,0.25);
[H0,w]=freqz(h0,1,10000);
figure,plot(w/pi,abs(H0));
xlabel('\omega/\pi');
ylabel('amplitudska
karakteristika');
title('Prototip filter');
figure,plot(w/pi,20*log10(abs(H0)));
xlabel('\omega/\pi');

```

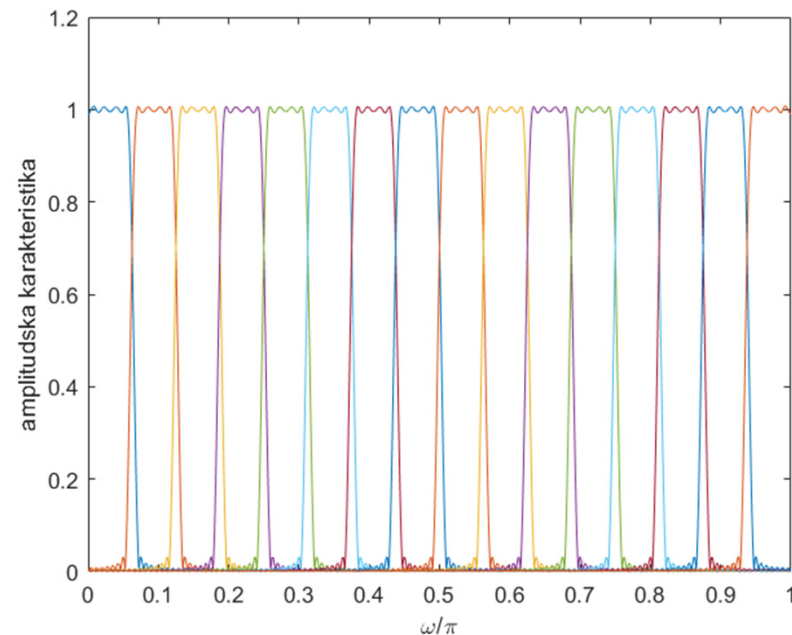


Kosinusna banka - primer

```

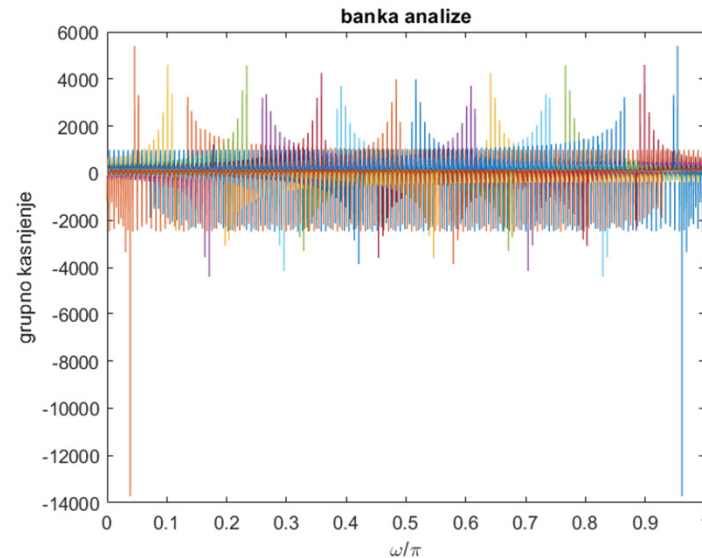
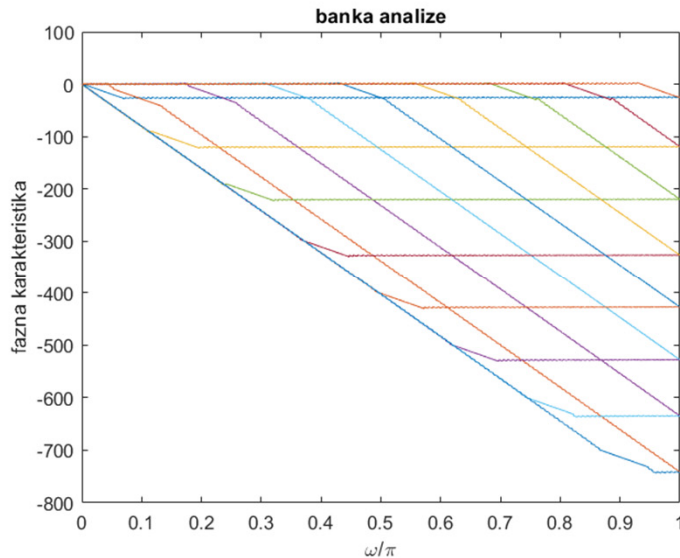
n=0:N;
for brojac=1:M
    h(brojac,:)=2*h0.*cos((brojac-1+1/2)*(n-N/2)*pi/M+(-1)^(brojac-1)*pi/4);
    [H(brojac,:),w]=freqz(h(brojac,:),1,10000);
    H_faza(brojac,:)=unwrap(angle(H(brojac,:)));
    H_grp_kasnjenje(brojac,:)=grpdelay(h(brojac,:),1,10000);
    g(brojac,:)=2*h0.*cos((brojac-1+1/2)*(n-N/2)*pi/M-(-1)^(brojac-1)*pi/4);
    [G(brojac,:),w]=freqz(g(brojac,:),1,10000);
    G_faza(brojac,:)=unwrap(angle(G(brojac,:)));
    G_grp_kasnjenje(brojac,:)=grpdelay(g(brojac,:),1,10000);
end
figure,plot(w/pi,abs(H)); hold on
xlabel('\omega/\pi');
ylabel('amplitudska karakteristika');
title('banka analize');

```



Kosinusna banka - primer

```
figure,plot(w/pi,H_faza);
xlabel('\omega/\pi');
ylabel('fazna karakteristika');
title('banka analize');
figure,plot(w/pi,H_grp_kasnjenje);
xlabel('\omega/\pi');
ylabel('grupno kasnjenje');
title('banka analize');
```

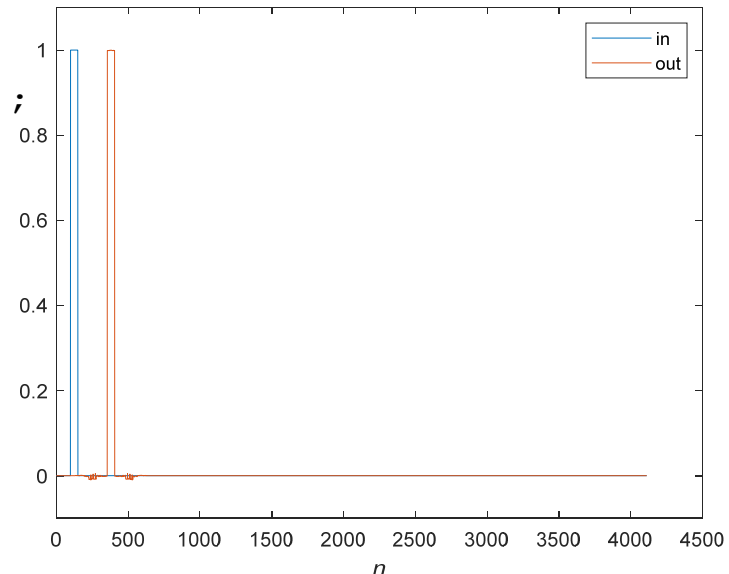


Kosinusna banka - primer

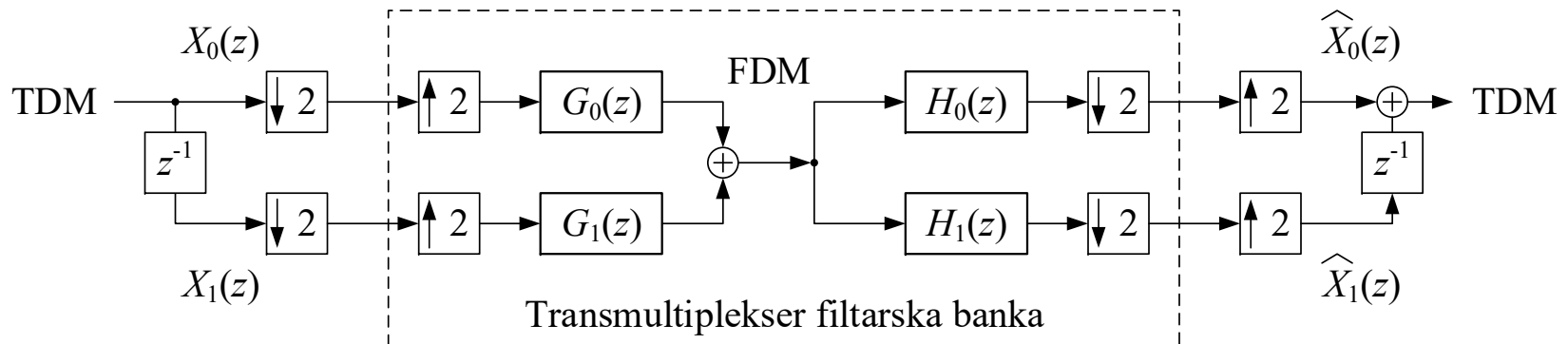
```

x=zeros(Nx,1);
x(100:150)=1;
Nx=length(x);
for br=1:M
    analiza_out_tmp=filter(h(br,:),1,x);
    analiza_out_d(1:Nx/M,br)=analiza_out_tmp(1:M:end);
end;
sinteza_in_d=analiza_out_d;
sinteza_out_d=filter(g(1,:),1,upsample(sinteza_in_d(:,1),M));
for br=2:M
    sinteza_out_tmp=filter(g(br,:),1,upsample(sinteza_in_d(:,br),M));
    sinteza_out_d=sinteza_out_d+sinteza_out_tmp;
end;
sinteza_out_d=M*sinteza_out_d;
figure,plot(0:Nx-1,x,0:Nx-1,sinteza_out_d);
ylim([-0.1 1.1]);
xlabel('\itn');
legend('in','out');

```



Transmultiplexeri



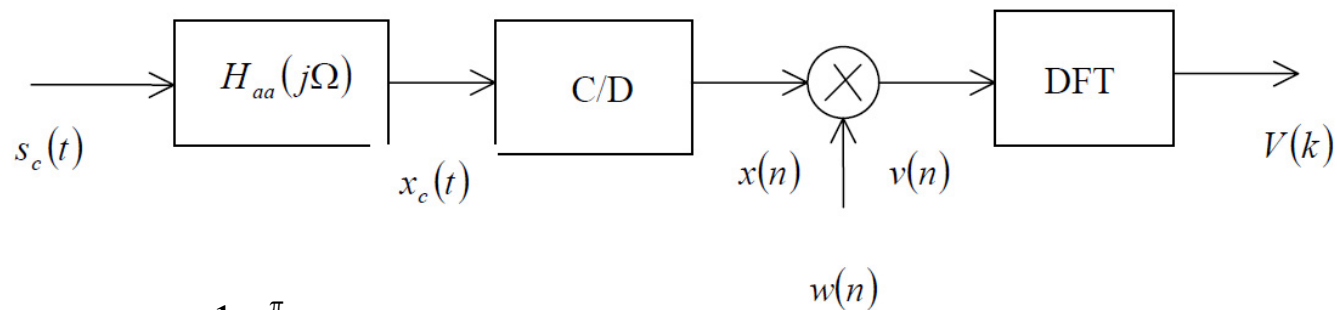
Dualni problem u odnosu na pojam filtarske banke koji smo do sada razmatrali

Prvo ide banka sinteze pa zatim banka analize

Wavelet transformacija

- Wavelet – talasić
- Wavelet transformacija je metod kojim se signal „razbija“ na niz komponenti (a nisu sinusne i kosinusne)
- U porođenju s STFT daje bolje rezultate za „prirodne“ signale jer su mogućnosti što se rezolucije tiče povoljnije za te klase signala
- U kontekstu filtarskih banaka, izborom tipa waveleta biramo zapravo koeficijente filtera i njihov međusobni odnos (tj. tip komplementarnosti)

Spektralna analiza - podsetnik



$$V(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) W(e^{j(\omega-\theta)}) d\theta$$

$$V[k] = V(e^{j\omega}) \Big|_{\omega = \frac{2\pi k}{N}}$$

$$V[k] = \sum_{n=0}^{N-1} v[n] e^{-\frac{2\pi kn}{N}}, \quad k = 0, 1, \dots, N-1$$

Vremenski zavisna Furijeova transformacija

Real-time spektralna analiza signala, zapravo se bazira na algoritmu vremenski zavisne Furijeove transformacije (***Time Dependant Fourier Transform*** ili ***Short Time Fourier Transform STFT***) koja je definisana formulom:

$$X(n, e^{j\omega}) = \sum_{m=-\infty}^{m=\infty} x[m] w[m - nR] e^{-j\omega m}$$

gde je:

x - ulazni signal,

w - prozorska funkcija (konačne dužine),

ω - frekvencija za koju se računa Furijeova transformacija,

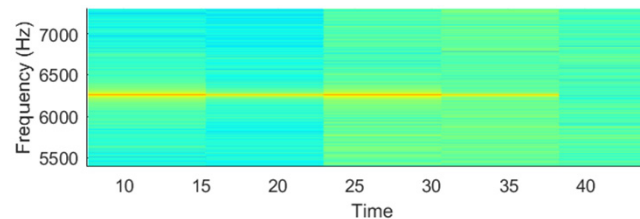
R – pomeraj početka dva sukcesivna prozora.

Spektrogram

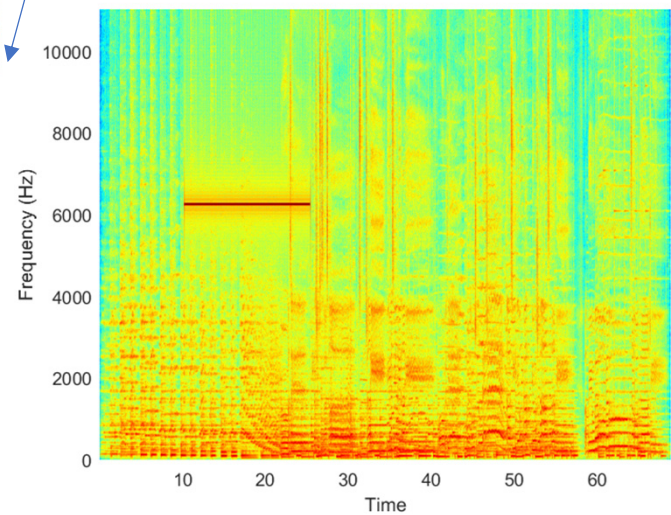
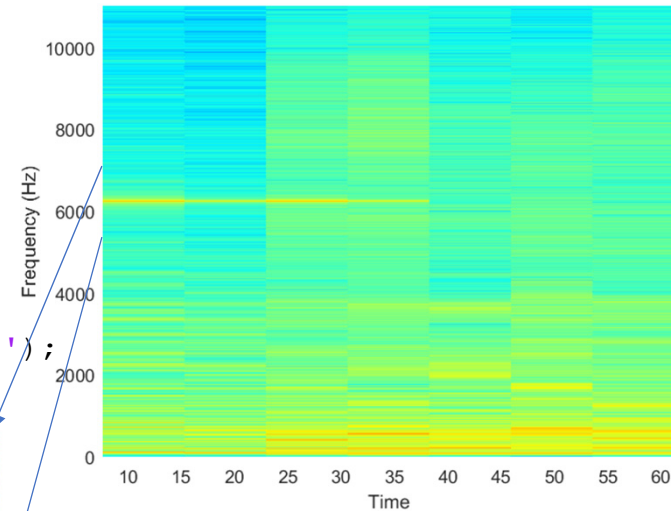
- Pogodan način da se prikažu rezultati
- Praktično se prikazuje matrica dobijena primenom vremenski zavisne Furijeove transformacije

Spektrogram

```
[S,F,T,P]=spectrogram(x);
figure,surf(T/(fs/pi/2),F/pi*fs/2,10*log10(P),'EdgeColor','none');
axis xy; axis tight; colormap(jet); view(0,90);
xlabel('Time');
ylabel('Frequency (Hz)');
```

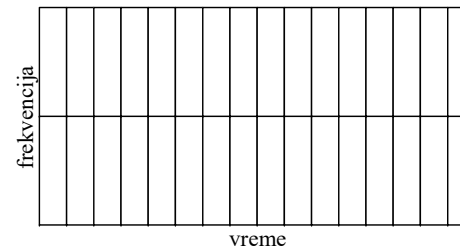
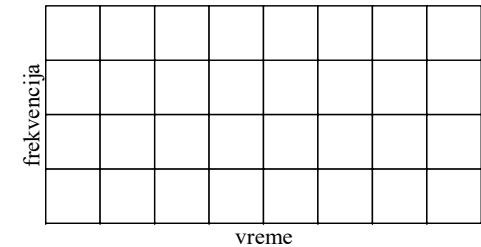
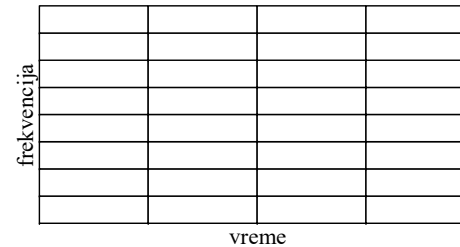


```
d_p=round(0.05*fs)+1;
nfft=1024*4;
[S,F,T,P]=spectrogram(x,d_p,0.5*d_p,nfft,fs);
figure,surf(T,F,10*log10(P),'EdgeColor','none');
axis xy; axis tight; colormap(jet); view(0,90);
xlabel('Time');
ylabel('Frequency (Hz)');
```



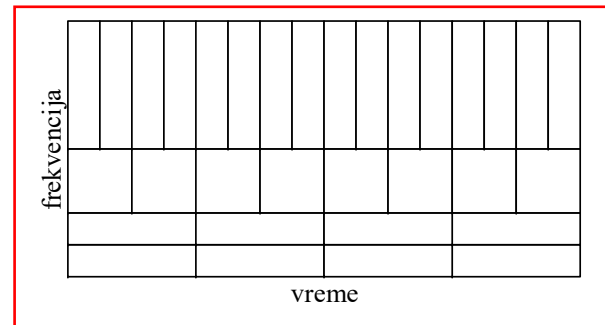
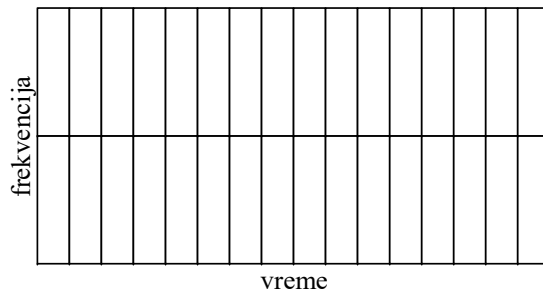
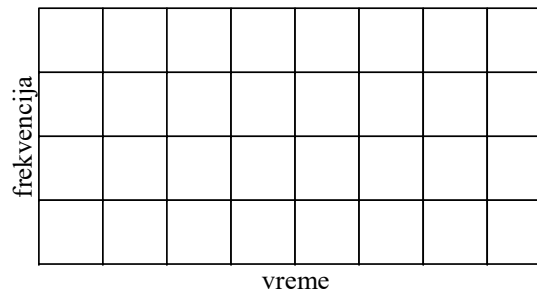
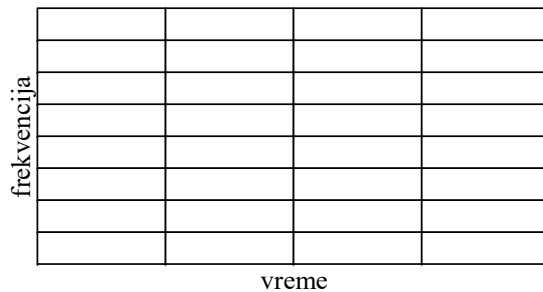
Izbor dužine prozora

- Posebno važno za nestacionarne signale
- Ako je dužina prozora N veća, popravlja se rezolucija po frekvenciji $\sim fs/N$, ali se kvvari rezolucija u vremenu (kratkotrajne, brze promene u signalu će biti izgubljene, tj. na neki način usrednjene)
- Za malo N rezolucija po frekvenciji je loša

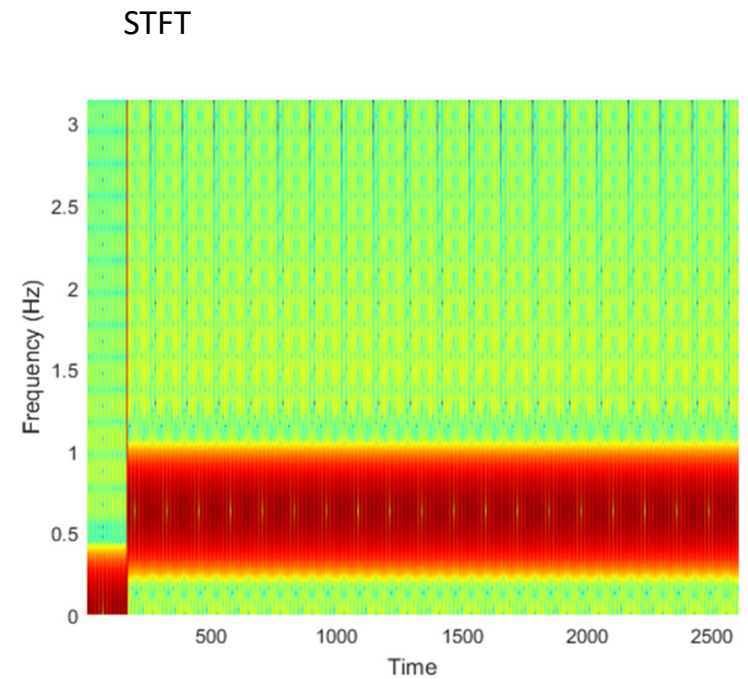
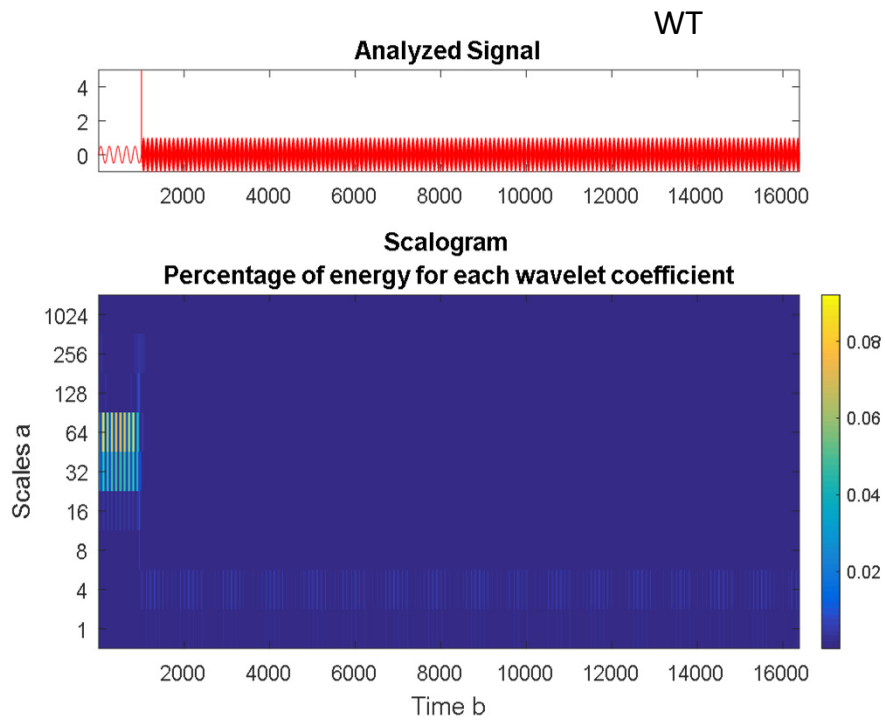


Izbor dužine prozora

Wavelet analiza omogućav drugačiju vremensko/frekvencijsku podelu



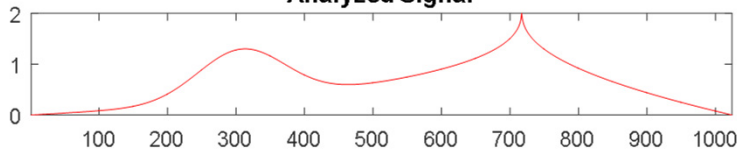
Primer



Primer

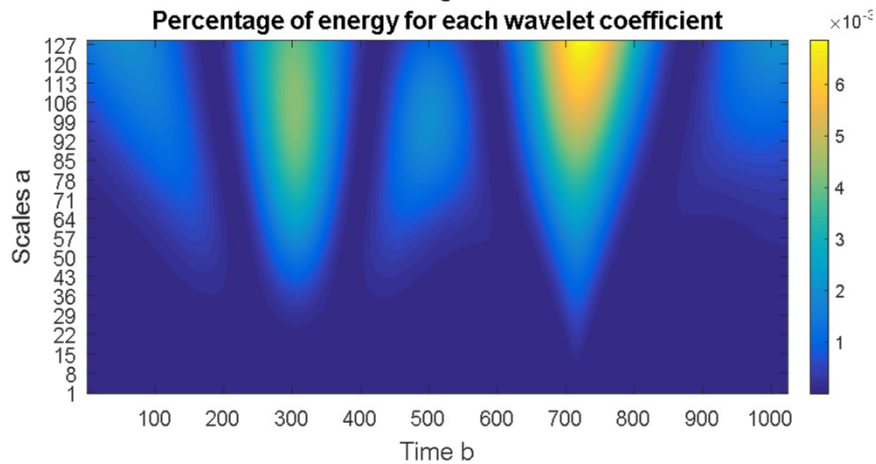
WT

Analyzed Signal

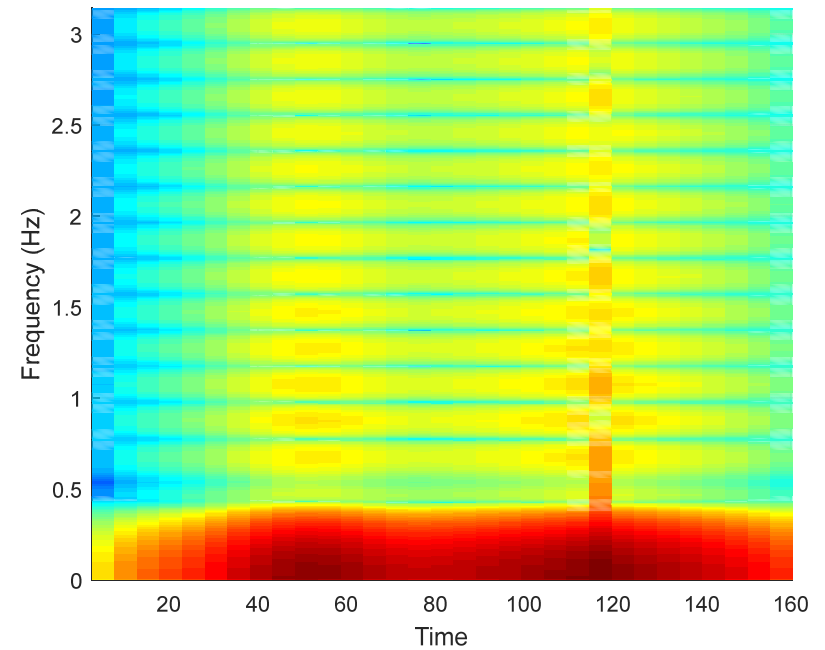


Scalogram

Percentage of energy for each wavelet coefficient



STFT



Kontinualna wavelet transformacija

$$CWT_x^\Psi = \Psi_x^\Psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \Psi^* \left(\frac{t-\tau}{s} \right) dt$$

$$CWT_x^\Psi = \Psi_x^\Psi(\tau, s) = \int x(t) \Psi_{\tau, s}^*(t) dt$$

$$\Psi_{\tau, s}^*(t) = \frac{1}{\sqrt{|s|}} \Psi \left(\frac{t-\tau}{s} \right)$$

Translacija – τ – mera vremena
 Scale – s – mera frekvencije –
 veća vrednost odgovara nižoj
 frekvenciji

Transformacija je niz
 konvolucija sa skaliranom i
 pomerenom prototip
 funkcijom (mother wavelet)

Diskretna wavelet transformacija

Parametar s dobija diskrente vrednosti, tipično 2, 4, 8...

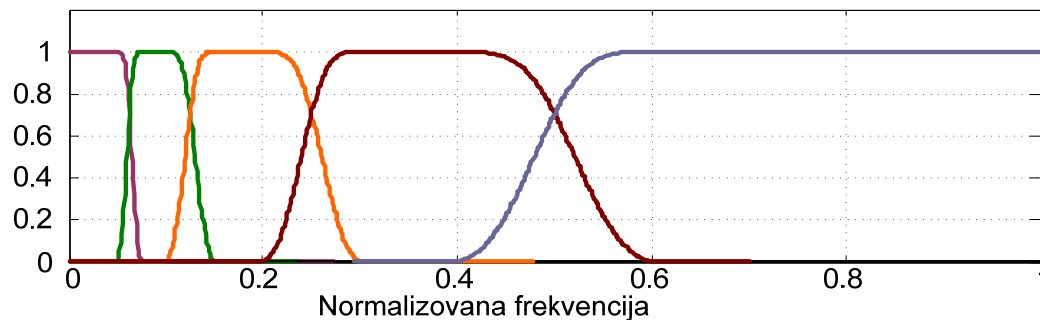
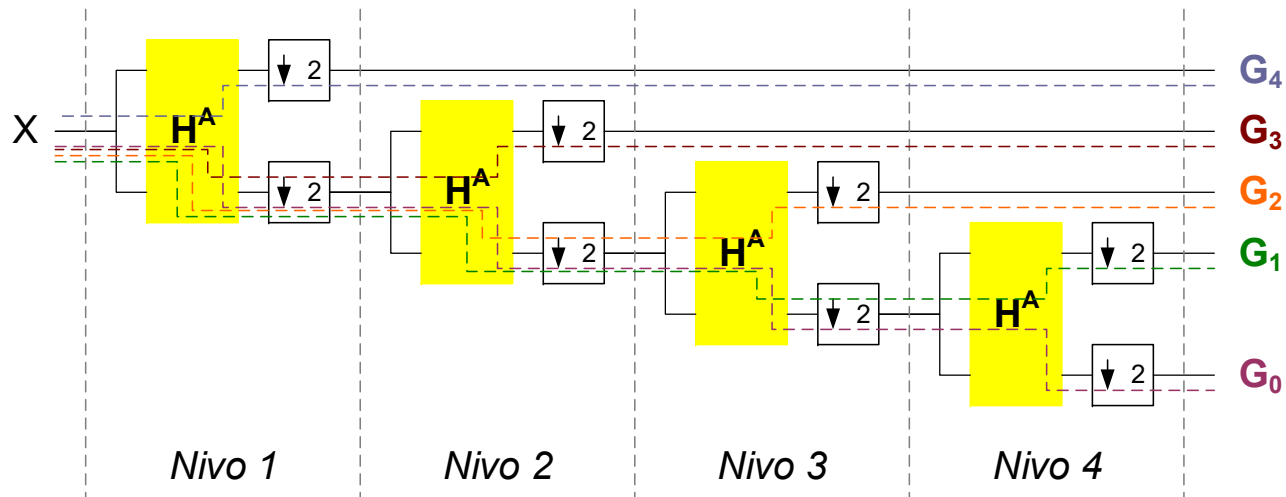
U implementaciji se svodi na oktavnu filtarsku banku sa savršenom rekonstrukcijom, gde se filtri banke analize biraju tako da odgovaraju nekoj određenoj wavelet transformaciji

Oktavne filtarske banke – tree struktura

- Realizovane preko dvokanalnih banaka
- U slučaju wavelet transformacije, parametri filtra zavise od tipa transformacije
- Na svakom nivou dobija se signal „aproksimacije“ i signal „detalja“
- Poslednja aproksimacija predstavlja LP (najniži u spektru) deo signala
- Signal se rekonstruiše prolaskom kroz banku sinteze
- Praktično, poslednja aproksimacija i svi detalji daju signal (uz odgovarajuće filtre u banci sinteze)

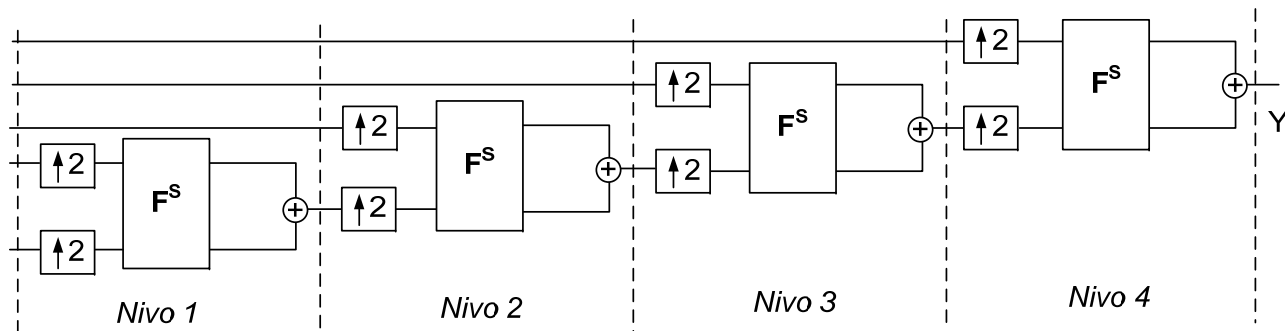
Oktavna banka analize

(realizovana pomoću dvokanalnih banki)



Oktavna banka sinteze

(realizovana pomoću dvokanalnih banki)



Dvokanalne banke sa savršenom rekonstrukcijom

- Do sada smo pomenuli ortogonalne filtarske banke (N_0 – red filtra)

$$H_1(z) = -z^{-N_0} H_0(-z^{-1})$$

$$F_0(z) = 2H_1(-z), \quad F_1(z) = -2H_0(-z)$$

- Gde je filter H_0 dobijen faktorizacijom polinoma, kao faktor minimalne faze filtra linearne faze (dobijenog od proizvoljnog filtra linearne faze „podizanjem“ i skaliranjem amplitudske karakteristike)

$$H_{LPHB} = H_0(z) H_0(z^{-1})$$

Wavelet transformacija kao oktavna filtarska banka

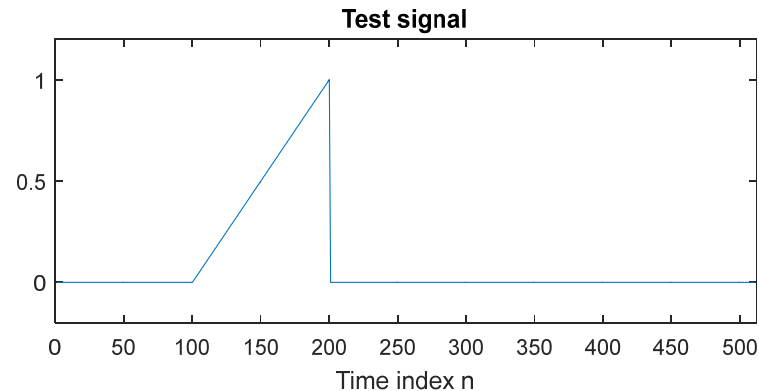
- U principu se različiti tipovi transformacije dobijaju izborom filtra H_0 i/ili različitim faktorizacijama

$$H_{LPHB}(z)$$

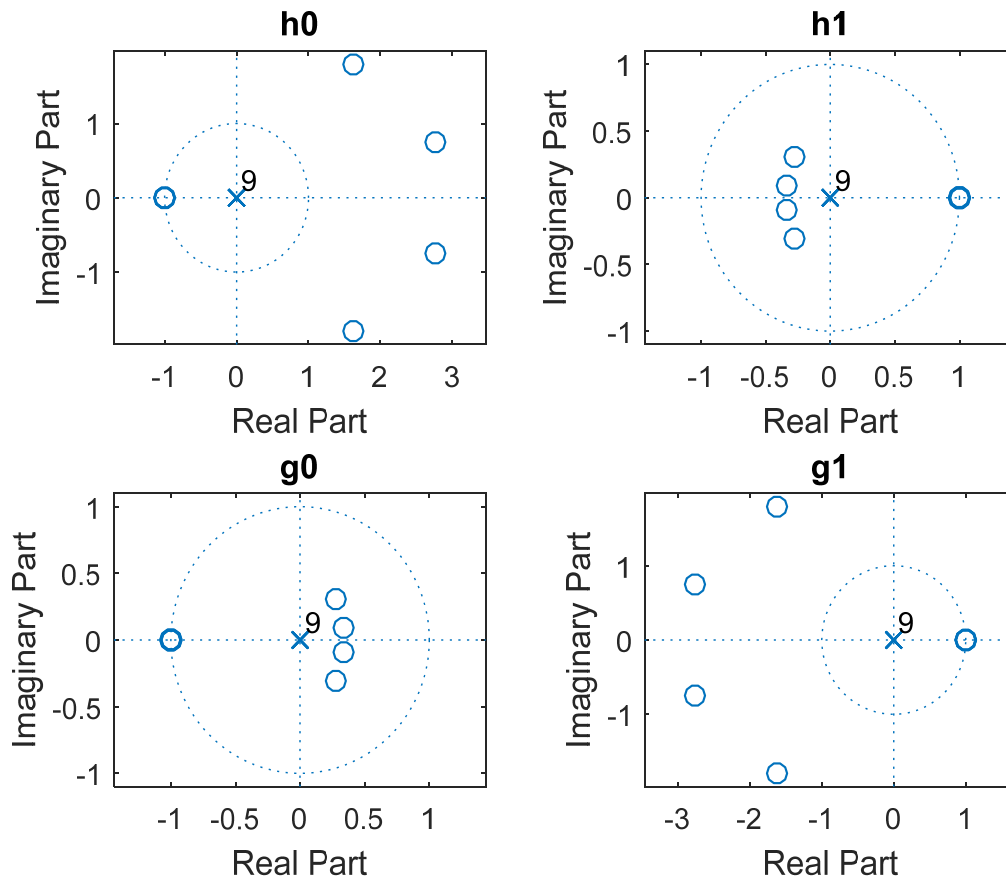
- Na primer, ako se faktorizacija uradi tako da faktori ostanu linearne faze dobija se biortogonalna filtarska banka, kod koje ne postoji simetrija u karakteristici filtera, ali su filteri u banci analize linearne faze

Primer 1

```
clear all close all
wname = 'db5'; % Set wavelet name. % Compute the four filters associated with wavelet
name given by the input string wname.
[h0,h1,g0,g1] = wfilters(wname); N = 10; % The length of db5 filter
% Test signal x=[zeros(size(1:100)),0.01*(1:100),zeros(size(201:511))]; figure (1)
subplot(3,1,1) plot(x) title('Figure 1: Test signal') xlabel('Time index n') axis([0,512,-
0.2,1.2]) % Signal analysis %
```

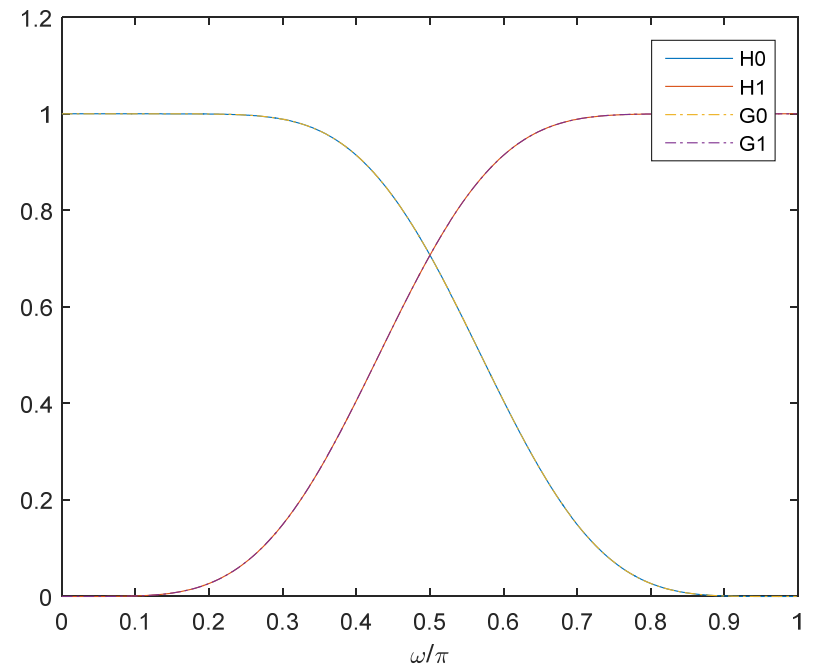
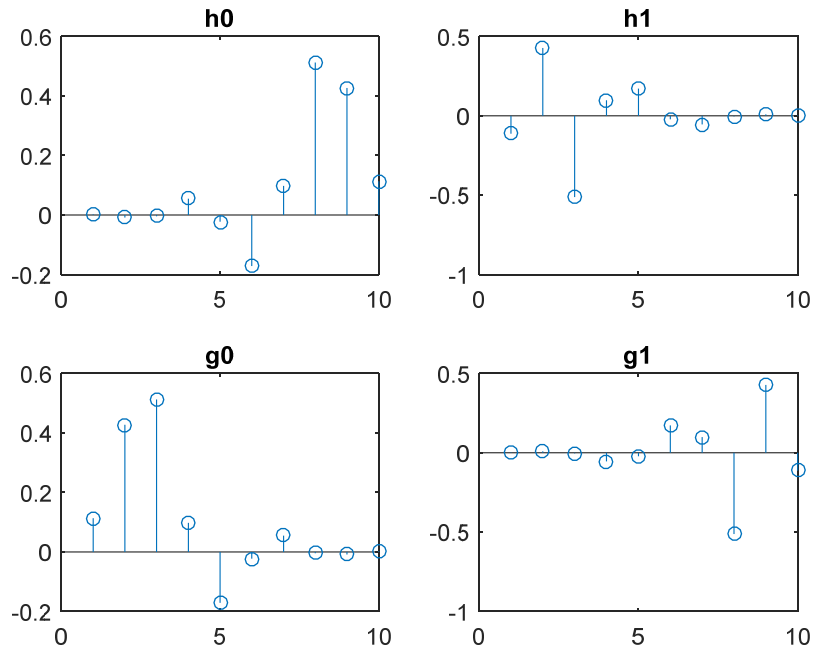


Primer 1

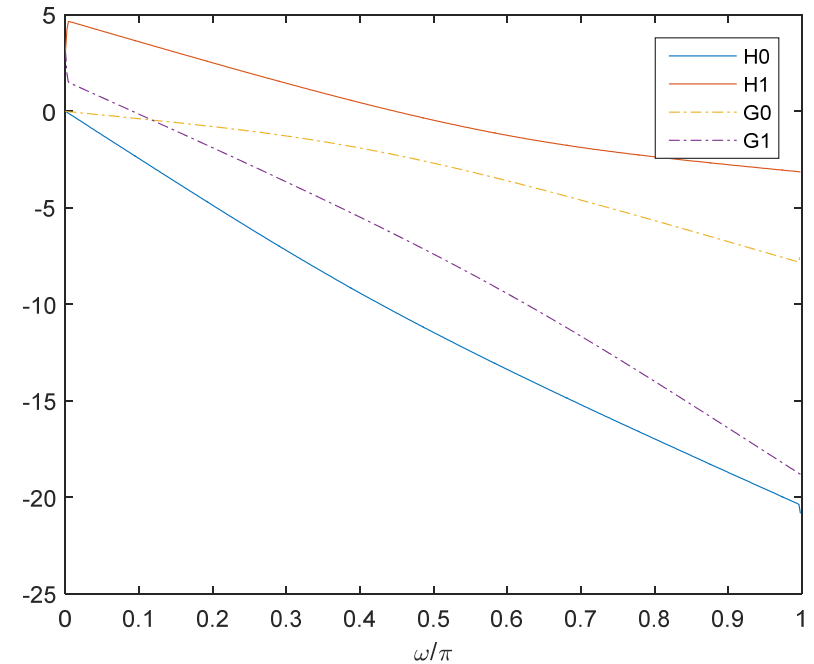
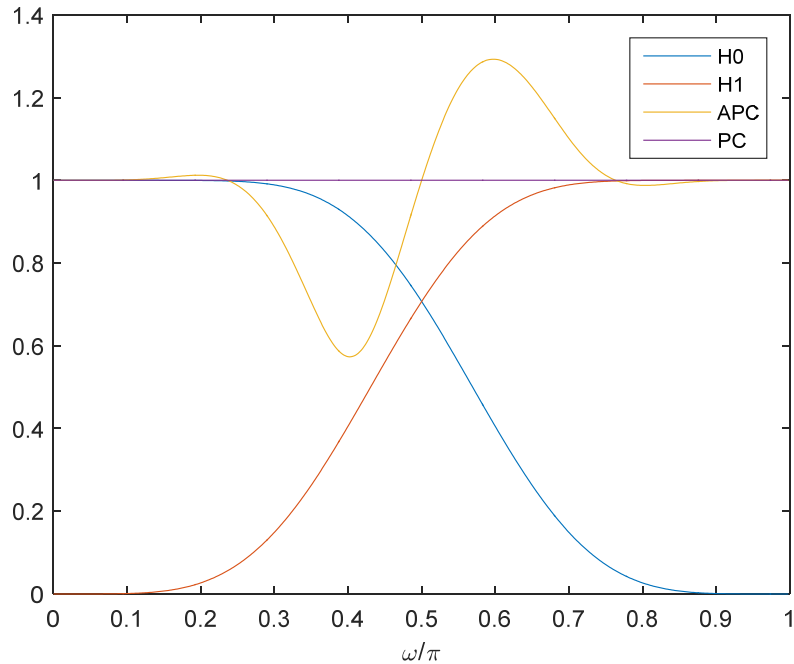


h0 lp maksimalne
faze

Primer 1

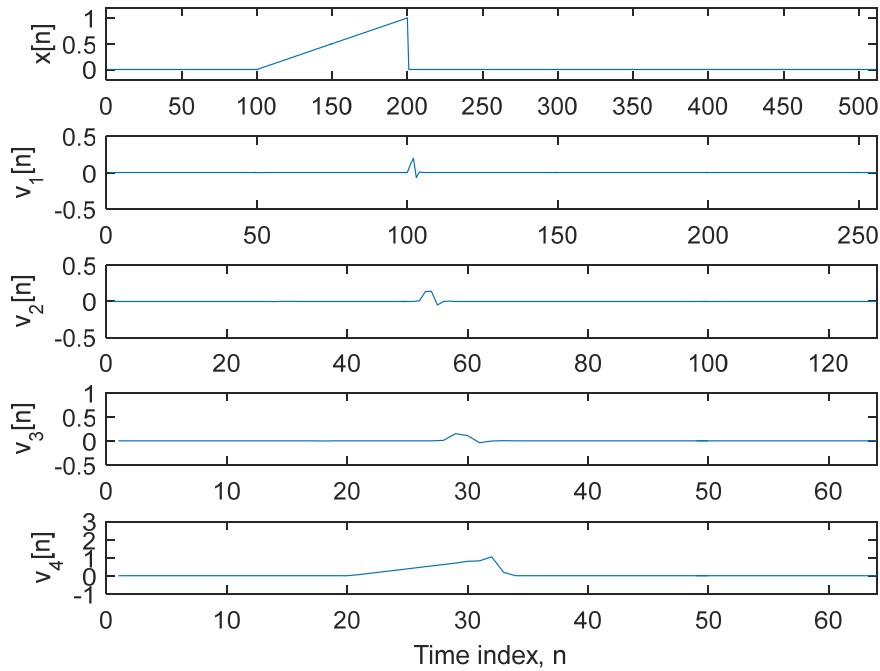


Primer 1



Filtri u banci analize su komplementarni po snazi ali nisu linearne faze

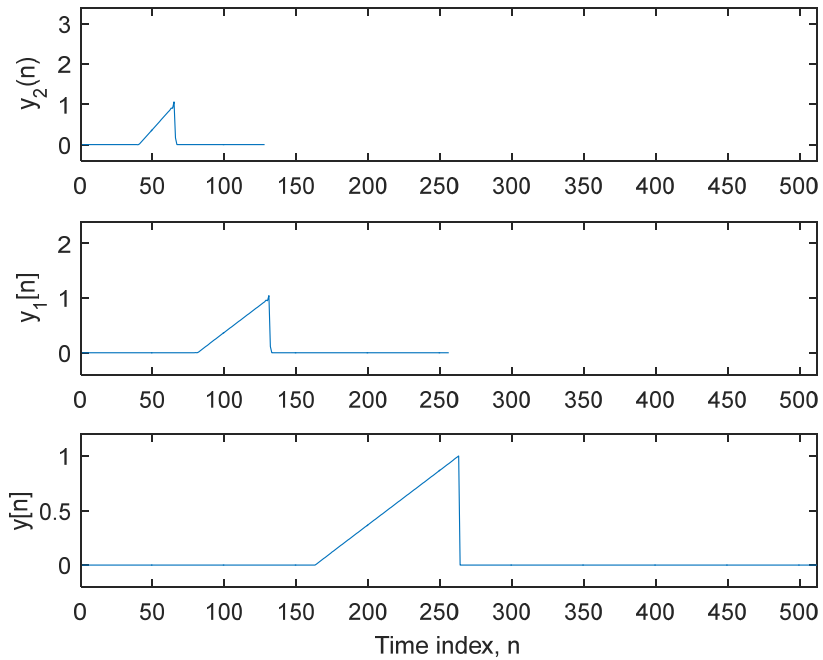
Primer 1



D
E
T
A
I
L

aproksimacija

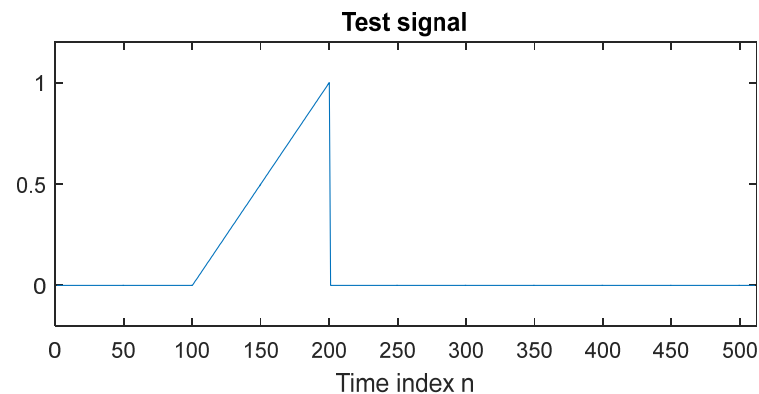
Primer 1



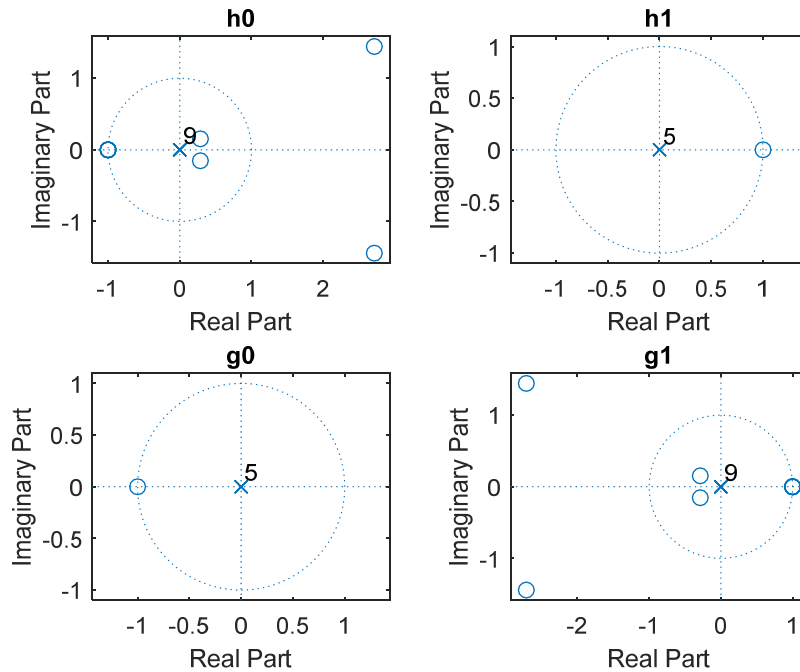
Savršena
rekonstrukcija

Primer 2

```
clear all close all
wname = 'bior1.5'; % Set wavelet name.
% Compute the four filters associated with wavelet name given by the input string
wname.
[h0,h1,g0,g1] =wfilters(wname);
N=length(h0);
x=[zeros(size(1:100)),0.01*(1:100),zeros(size(201:511))]; figure (1) subplot(3,1,1) plot(x)
title('Figure 1: Test signal') xlabel('Time index n') axis([0,512,-0.2,1.2]) % Signal analysis %
```

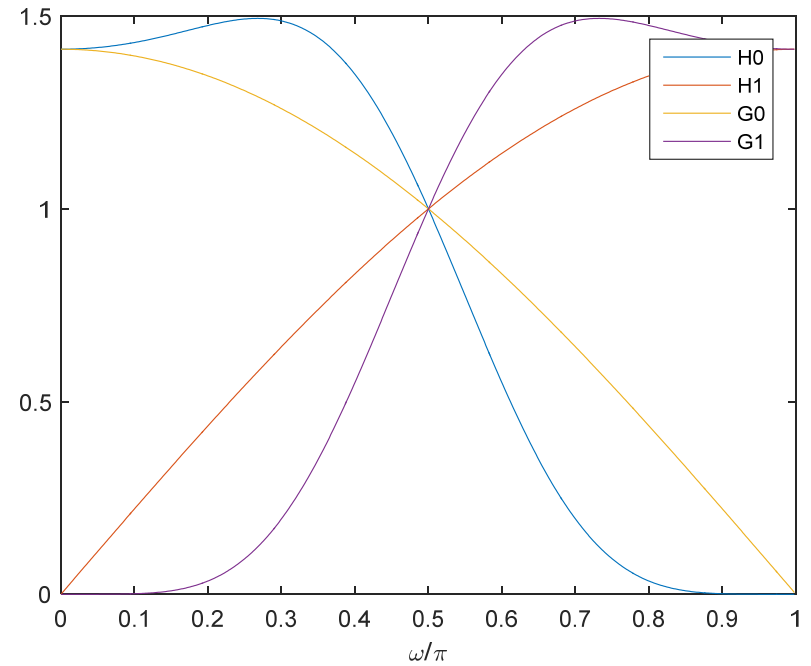
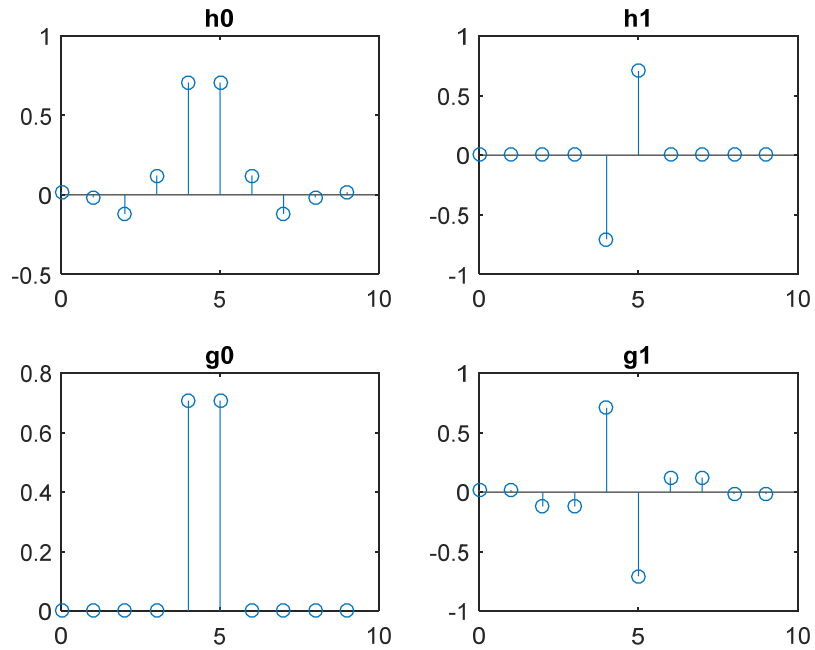


Primer 2

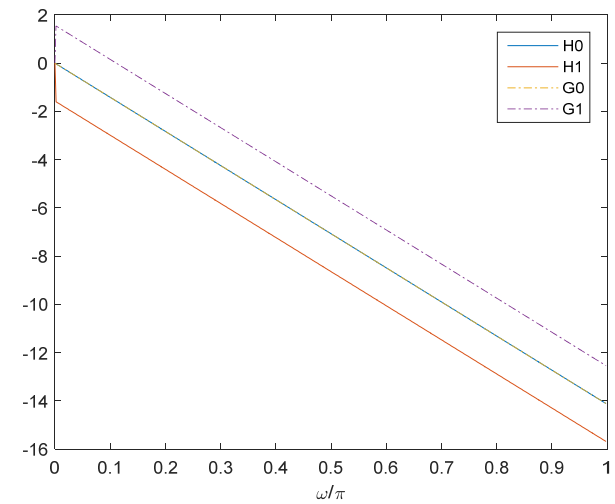
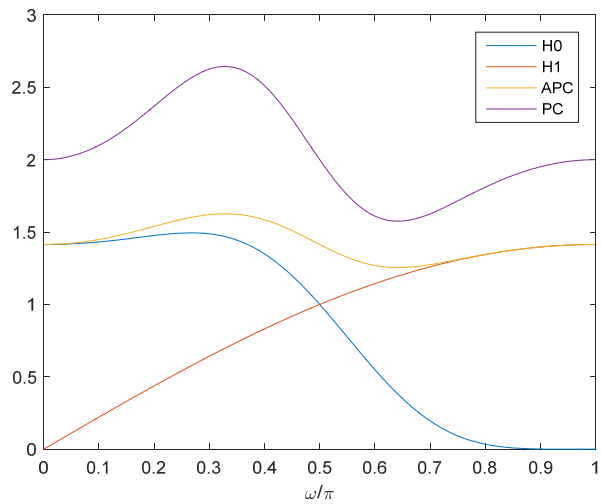


h0 Ip linearne
faze

Primer 2

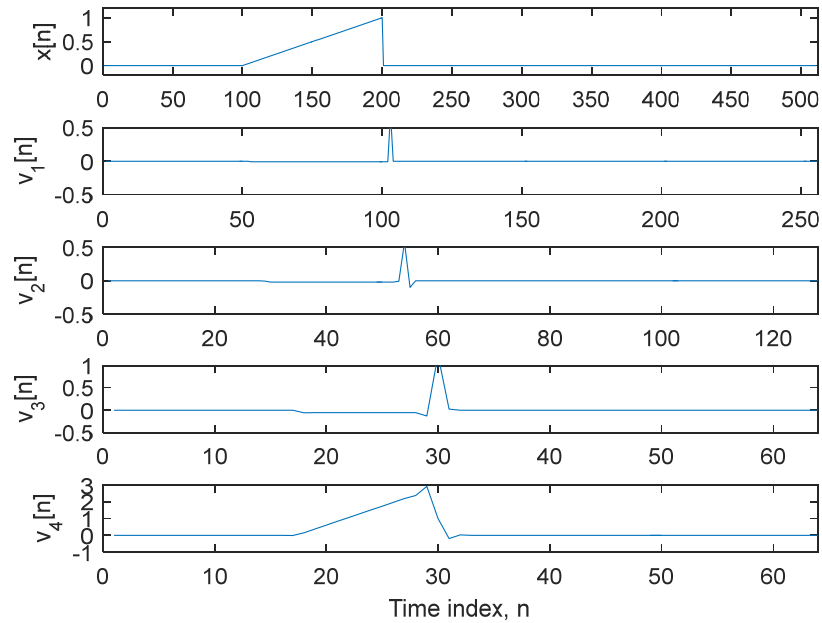


Primer 2



Filtri u banci analize nisu komplementarni ali su linearne faze

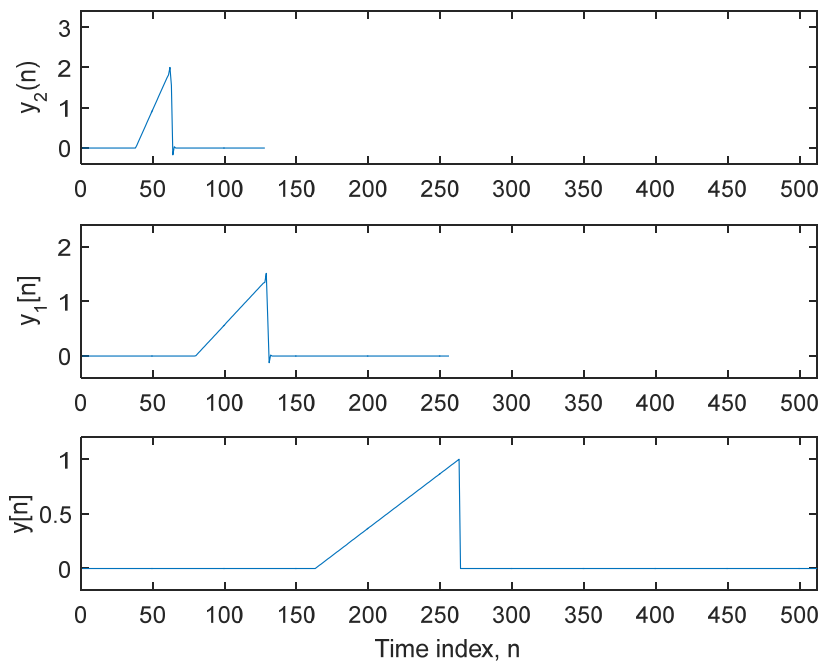
Primer 2



D
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aproksimacija

Primer 2



Savršena rekonstrukcija